CEOC

Centro de Estudos em Optimização e Controlo
(Centre for Research in Optimization and Control)

Computability and Algorithms

Research Plan 2003-2005

Universidade de Aveiro
Departamento de Matemática

Research Plan 2003-2005

1. Computability and algorithms

1.1 Research team

- **Coordinator**: Maria Rosália Dinis Rodrigues
- **Research Team**:
  1. Maria Rosália Dinis Rodrigues (PhD, 30%)
  2. Antonio Leslie Bajuelos Dominguez (PhD, 25%)
  3. António Ferreira Pereira (PhD Student, 100%)
  4. Ana Mafalda Martins (PhD student, 70% em 2006)
  5. Inês Pereira de Matos (PhD student, 100%)
- **Scientific cooperation**:
  1. Ana Maria Carvalho de Almeida, Universidade de Coimbra
  2. Ana Paula Tomás, Universidade do Porto
  3. Gregório Hernandez Peñalver, Universidad Politécnica de Madrid
  4. Manuel Abellanas Oar, Universidad Poltécnica de Madrid, Spain

1.3 Project's Summary

This project is situated in the general area of Theoretical Computer Science. It comprises the work developed by two senior researchers and four PhD students (two to start in 2004/2005) from CEOC and other R&D units. The ongoing research concerns the topics of: Complexity of Problem Instances, Algorithms and Heuristics for Computational Geometry and Quantum Computation.

1.4 Project's Description Objectives

1. **Overall Objectives**. The underlying motivation for this project is the establishment of a solid basis for the creation of a research group in the Computational Mathematics area. There are several reasons for this resolution: the University of Aveiro has stated the need for the development of Informatics and Computer Science research groups, the Mathematics Department has expressed the need for research in Computer Science and the FCT's Mathematics Evaluation Panel has regularly pointed out the need for research in Computational Mathematics within portuguese R&D units.

   We believe that the existence of this project can play an important role in the motivation and recruitment of young researchers in this area.

2. **Specific Objectives**.

   - Objectives on Complexity Theory: the search for a theoretical framework for the classification of the Instances of NP-hard Problems, in terms of their computational cost.
   - Objectives on Computational Geometry: to develop approximation algorithms and heuristics for the Minimum Vertex Guard Problem, to study de Vertex π-Floodlights Problem for Orthogonal Polygons and to characterize the visibility graphs of certain classes of polygons.
   - In the field of Quantum Computation, our work focus on the design of quantum arithmetic circuitry, particularly on the use of generalized redundant number systems, as well as on the problem of quantum computing simulation.

1.5 Relationship with the state of the art

1. **Complexity Theory**. Although an enormous amount of work has been published on the classification of problems in terms of their Computational Complexity, very little is know about the comparative behaviour of particular instances of a given problem of the NP classes.

2. **Computational Geometry**. In 1973, Victor Klee posed the following questions: How many guards are necessary and how many are sufficient to patrol the paintings and works of art in a gallery with n walls? Since publication of the first original result (*Chvátal Art Gallery Problem*), tremendous amount of research on illumination problems has been carried out by mathematicians and computer scientists. In 1981 the Minimum Vertex Guard Problem, i.e. the problem of finding the minimum number of guards needed to cover any polygon, was proved to be NP-hard. In the Handbook of Computational Geometry (Elsevier, 2000), Urrutia asserts that one approach that has been neglected in the study of Art Gallery Problems is the one of finding algorithms that obtain approximate solutions. The most

well-know result on this subject is an algorithm that finds in $\mathcal{O}(n^5 \log n)$ a vertex guard set that is at most $\mathcal{O}(\log n)$ times the minimum number of vertex guards needed.

3. **Quantum Computation**.

   - **Generalized redundant number systems in quantum computation.** Last two decades have brought many and significant advances in both theoretical and practical understanding of computation in qubit systems. However, the development of a theoretical framework for quantum computation in qudit systems started only recently. Universal quantum gates for hybrid quantum systems and quantum codes are hot topics in the area.
   - **Design of quantum arithmetic circuitry.** Considerable effort is being devoted to the building of a scalable quantum processor, using a wide range of competing technologies with several promising results. The design and complexity analysis of quantum circuits for arithmetic operations is an important and open research area.
   - **Quantum computer simulation.** Quantum computer simulators are imperative at present, since quantum hardware prototypes hardly exist. A variety of quantum simulators is either available or under development. However, the execution time and memory requirements of these simulators increase exponentially with the number of qubits which constrains the size of problem instances to be solved.

1.6 Expected indicators

Number of Publications	**2003**	**2004**	**2005**	**Total**
Books	0	0	1	1
Papers in international journals	2	5	7	14
Papers in national journals	1	0	1	2
Number of Communications	**2003**	**2004**	**2005**	**Total**
in International Meetings	2	3	6	11
in National Meetings	0	0	0	0
Reports	0	6	8	14
Organization of seminar and conferences	3	6	7	16
Advanced training	**2003**	**2004**	**2005**	**Total**
number of PhD theses	0	1	0	1
number of Master theses	0	2	4	6

1.7 Tasks

1.7.1. Complexity Theory

- **Task duration - months**: 36

- **persons*month**: 3,6
  - **Research team**: Maria Rosália Dinis Rodrigues (10%).
- **Expected results**: The completion of a model, presently being developed, based on the concepts of Algorithmic (or Kolmogorov) Complexity, Information Theory and Maximum Entropy Principle, for the computational evaluation of the complexity of particular problem instances.
- **Task description**:
  - To refine the existing model for the computational evaluation of the complexity of problem instances.
  - To derive efficient Entropy estimators.
  - To establish a basis for a complete Classification of instances of NP-hard problems.
  - To search for a relationship between known classes of heuristics and the classes of instances derived from the model.

1.7.2. Computational Geometry

- **Task duration - months**: 36
- **persons*month**: 9
  - **Research team**: Antonio Leslie Bajuelos Dominguez (25%).
- **Expected results**:
  - The development of Approximation Algorithms and Heuristics for the Minimum Vertex Guard Problem.
  - Identification of structural properties of certain classes of polygons which might prove relevant for the building of heuristics, as well as the characterization and recognition of the Visibility Graph.
  - Development of auxiliary tools and random generators for geometric structures.
  - Application of parallel processing techniques to the solving of illumination problems.
  - Implementation and assessment of the developed methods.
- **Task description**: We intended to study and develop algorithms and heuristics for the visibility and illumination problems. Most of these problems belong to the NP classes. Two aspects must be considered. The first one is the identification of structural properties of certain classes of polygons and terrains which might prove relevant for the building of heuristics for the resolution of the Minimum Vertex Guard Problem, that is the problem of finding the minimum number of guards needed to cover any

polygon. On the other hand, we intend to study some visibility and illumination problems that still remain open, e.g. Visibility Graph Recognition and Vertex π-Floodlights Problem.

1.7.3. Quantum Computation

- **Task duration - months**:
  - **Research team**: Maria Rosália Dinis Rodrigues (20%), António Ferreira Pereira (100%).
- **Expected results**:
  - **Generalized redundant number systems in quantum computation.** To investigate the implications of redundant integer encodings as qudit states in the complexity of search problems.
  - **Design of quantum arithmetic circuitry.** To design efficient quantum arithmetic circuits.
  - **Quantum computer simulation.** To develop a full working package in *Mathematica* for the symbolic simulation of quantum computation. To evaluate the results of this new method, when compared with known quantum simulators.
- **Task description**:
  - **Generalized redundant number systems in quantum computation.** Existing methods assume a non-redundant encoding of integer digits as states in the canonical basis of the d-dimensional Hilbert space associated with each qudit. Redundant numbers systems have widely been used in the development of very efficient classical solutions of many problems, e.g. in digital arithmetic algorithms, but only a few and localized applications of redundancy in quantum algorithms are known.

    The use of redundancy changes the space structure of search problems. Our goal is to analyze these changes, from a computational complexity viewpoint.
  - **Design of quantum arithmetic circuitry.** A sound universal set of redundant quantum gates still needs to be established, as well as the subsequent design of efficient quantum circuits for redundant arithmetic.
  - **Quantum computer simulation.** The development of specific simulators for quantum computation based on symbolic and algebraic manipulation of states and operators is a new topic of research with several important practical applications. Our task is to implement symbolic algebraic software for sub-exponential simulations of quantum algorithms for *medium* size problem instances.