CTS Research Project for Ilona Dzenite

Home Institute: Riga Technical University, Latvia

CTS Host Institute: University of Aveiro, Portugal

Title of the Research Project: Impedance Change and the Calculus of Variations

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CTS stay: 6 months (January 2005 - June 2005)

1. Introduction

Non-destructive testing methods are widely used in industry for quality control of products and materials. The term “non-destructive testing” usually refers to inspection methods for testing properties of materials and the quality of products without damaging or impairing the test objects.

Since many high-technology devices operate under extreme conditions in temperature and pressure, or come into prolonged contact with chemically active materials, etc., it is important to develop non-destructive testing methods, which ensure their safety and reliability. In fact, a central objective of non-destructive testing is to decide whether a device (or material) can successfully perform specified functions. Perfect devices or materials are rare. From a practical point of view, a material is considered to be of good quality if its parameters lie within specified tolerances. Therefore, the purpose of non-destructive testing can be formulated in the following way: to determine whether the relevant parameters of a material (or characteristics of a device) lie within prescribed limits.

Among the non-destructive testing methods in use today, are the following: X-rays, Messbauer analysis, neuron activation, ultrasound, acoustic emission, microwaves, dielectric spectroscopy, and eddy currents.

The eddy current method is based on the law of electromagnetic induction. Therefore, eddy current testing devices are widely used for the quality control of electrically conducting objects, such as metals, alloys and semiconductors. In industry eddy current devices are used, for example, to control the size of products, to measure the diameter of wires and tubes, the thickness of walls and metal sheets. They are widely used to control the thickness of metal covering and the thickness of layers in multiplayer products. They can also be used to estimate the rate of destructive corrosion.

Another widespread field of applications of eddy current methods is the detection of flaws in materials: cracks, fiberings and non-metallic inclusions. Flaw detection is very important in the transport industry, including aircraft, ship and automobile. Eddy current methods are widely used in the nuclear industry and, in particular, to determine flaws ant the thickness of walls in heat exchanger tubes. An important application is the quality control of spot welding.

The idea of the eddy current method is as follows. Suppose that a coil carrying an alternating current is situated in the vicinity of a conducting medium to be tested. The current, passing through the coil (so-called excitation coil), generates a varying magnetic field. This magnetic field (so-called primary field) induces varying currents (eddy currents) in the electrically conducting medium according to the principle of electromagnetic induction. These currents, in turn, produce a varying magnetic field (so-called secondary field). The effects of secondary field can be seen from the variation of the output signal of the excitation coil or from the output signal of the second coil (so-called detector coil) situated nearby. In general, the output signal represents the resultant field, that is, the sum of the primary and secondary fields.

The output signal of the detector coil depends on several parameters, such as the magnitude and frequency of the alternating current, the electrical conductivity and magnetic permeability of the medium, as well as the relative position of the coil and the medium. It also reflects the presence of inhomogeneities (so-called flaws) in the medium. Eddy current flow changes the impedance of the coil. Therefore, it is important to study the basic principles of the interaction of eddy currents in non-uniform conducting media. The output signal of an eddy current probe is a function of several parameters. Eddy current method's strength is due to the fact that one can achieve "recognition" of flaws by selectively varying the different parameters. The method's weakness comes from the combined influence of several parameters, which may lead to an erroneous interpretation of the output signal. In this case, mathematical modelling can clarify the situation considerably by "freezing" all except one, or a few parameters, and observe the influence of the varying parameters on the eddy current probe output signal. This allows to <u>optimise the parameters</u> of an eddy current probe for <u>the control process</u> in hand.

Consider a problem on influence of a non-uniform conducting half-space, located in region $z<0$, on a single-turn coil carrying the alternating current. The single-turn coil of radius r_c is located in free space (region R_0) at the height h above the conducting half-space (region R_1) with conductivity σ_1. The conducting half-space contains a flaw (region R_F) with conductivity σ_F. In particular case of the axial symmetry, the complex-valued amplitude vector potential, has only one non-zero component, $\vec{A}=A(r,z)\vec{e}_\varphi$, and does not depend on φ. Problem for this component has the form (see [Antimirov, Kolyshkin & Vaillancourt , 1997]):

$$\begin{cases} \Delta_\varphi A_0 = -\mu_0 I^e, & in\ R_0, \\ \Delta_\varphi A_1 + k_1^2 A_1 = 0, & in\ R_1, \\ \Delta_\varphi A_F + k_F^2 A_F = 0, & in\ R_F, \end{cases} \tag{1}$$

where I^e is complex-valued external current density:

$$I^e = I\delta(r-r_c)\delta(z-h), \tag{2}$$

$\delta(y)$ is the Dirac delta function, $k_1^2=-j\omega\mu_0\mu_1\sigma_1$, $k_F^2=-j\omega\mu_0\mu_F\sigma_F$, $j=\sqrt{-1}$ is the imaginary unit, μ_0 is the magnetic constants, μ_1 and μ_F are the relative magnetic permeability in regions R_1 and R_F, respectively, ω is the frequency, and

$$\Delta_\varphi f(r,z) = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} - \frac{1}{r^2} f \ . \tag{3}$$

Boundary conditions at the interface between regions R_0 and R_1, and, regions R_1 and R_F, respectively, represent the condition of continuity of tangential components of the electrical and magnetic field vectors on the boundaries of these regions, and with the prescribed conditions at infinity.

In the case of the axial symmetry, the induced change in impedance has the form

$$Z^{ind} = \frac{j\omega}{I} \oint_C A_l^{ind} dl \,, \tag{4}$$

where A^{ind} is the induced vector potential (i.e. the contribution in the vector potential due to the presence of the conducting medium in region $z < 0$), C is the closed contour of the source of current.

2. Description of the Problem and Objectives

The first theoretical result in this area was presented in [Sobolev, 1963] for the case of a uniform conducting medium. In the case of the flaw of an arbitrary form, there does not exist an analytical solution. Therefore, since 1963, different approximate and numerical methods were developed for solving this type of problems. In this connection, the significant moment is a formula for Z^{ind} used in the literature (see [Auld, Muennemann & Wilson, 1981], [Auld, Muennemann & Riaziat, 1984], [Zaman, Gardner & Long, 1982], [Satveli, Moulder, Wang & Rose, 1996]). It has the form

$$Z^{ind} = -\frac{(\sigma_F - \sigma)}{I^2} \iiint_{V_F} \vec{E} \cdot \vec{E}_F \, dV, \tag{5}$$

where V_F is the region of the flaw, σ_F and σ are the conductivities of the flawed and flawless regions, respectively, $\vec{E}_F$ is the amplitude electric field vector in the flawed region, $\vec{E}$ is the amplitude electric field vector in the same region in the absence of the flaw.

As part of the PhD thesis of Ilona Dzenite, a new formula for Z^{ind} is obtained in the form

$$Z^{ind} = \frac{\omega^2 (\sigma_F - \sigma)}{I^2} \iiint_{V_F} \vec{A} \cdot \vec{A}_F \, dV, \tag{6}$$

where $\vec{A}_F$ is the amplitude vector potential in the flawed region, $\vec{A}$ is the amplitude vector potential in the same region in the absence of the flaw.

<u>The main goals of this project are:</u>

1) To prove the equivalence of Eqs. (5) and (6).
2) To obtain the formula for Z^{ind} in the case of n arbitrary form flaws.
3) To find $\vec{A}_F$ and $\vec{E}_F$. Since it is impossible to find $\vec{A}_F$ and $\vec{E}_F$ analytically, different approximate methods are to be used for determining these functions. In 1996 the method of Layer approximation was suggested in [Satveli, Moulder, Wang & Rose, 1996]. In this method, region of the flaw, V_F, is replaced by the flat horizontal layer with the same conductivity σ_F. The layer is located in region $z_1 \le z \le z_2$. Planes $z = z_1$ and $z = z_2$ are tangential planes to the region V_F. Such problem is solved analytically. Obtained solution for $\vec{E}_F$ of this problem is substituted into Eq. (5). A different approximate method - the method of additional currents – is developed in the candidate's PhD thesis. The main idea of the method is as follows. Let a flaw with conductivity σ_F be located in region V_F. Replacing the region containing the flaw with a uniform medium, the obtained problem can be solved analytically, and one can find the current density by equation $\vec{I} = -j\omega\sigma\vec{A}$.

If one assumes that there exists the current with density $-\vec{I} = j\omega\sigma\vec{A}$ in region V_F, after solving such problem analytically, the obtained solution, $\vec{A}_{new}$, can be substituted into Eq. (6).

4) To obtain an approximate formula for Z^{ind} in the case $\sigma = \sigma(M)$, by replacing $\sigma(M)$ with $\overline{\sigma} = \frac{1}{V_F}\iiint_{V_F}\sigma(M)\,dV$.

To reach the proposed objectives, the expertise of the Control Theory Group of the University of Aveiro on Computer Algebra Systems (see [Gouveia & Torres, 2004]), and on the Calculus of Variations (see e.g. [Torres, 2004a & 2004b]), will be crucial.

Computer Algebra Systems facilitate symbolic and numerical mathematics: (a) they use exact arithmetic and do not suffer from loss of precision or significance; (b) they work symbolically; (c) they are speedy, efficient and reliable tools for performing long and tedious calculations; (d) they provide an enormous gain in time and effort as far as mathematical analysis and synthesis is concerned. A CAS will be useful, for example, to verify the equivalence of Eqs. (5) and (6).

One of the effective methods for determining integral characteristics in the problems on heat convection (for example, Rayleigh critical number) are the Bubnov - Galerkin and Ritz methods used in the calculus of variations (see [Gershuni & Zhuhovicky, 1972]). Using even several basic functions in the calculus of variations, Rayleigh critical number differs from the exact one less than 1%. Since Z^{ind} is also an integral characteristics, it must be perspective to use the calculus of variations for obtaining Z^{ind}. The most important here will be selection of basic functions.

This project, submitted for a Marie Curie fellowship, will represent an important complement to the PhD research project of Ilona Dzenite.

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