CTS Research Project for Elena Ligere

Home Institute: Riga Technical University, Latvia

CTS Host Institute: University of Aveiro, Portugal

Title of the Research Project: Analytical solutions to some magnetohydrodynamical control problems on a flow of conducting fluid

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1. Introduction

Magnetohydrodynamics (MHD) studies the influence of the external magnetic field on the motion of a conducting fluid. Besides, MHD studies the appearance of a flow of conducting fluid due to the current passing through the fluid (so-called Electrically induced vortical flows).

Basic effects of MHD can be illustrated by example of a flow of viscous conducting fluid in a plane channel situated in a cross magnetic field.

We suppose that two walls of the channel $z = \pm b$ have the conductivity σ_w that is much larger than conductivity of the fluid and walls $y = \pm a$ are non-conducting. External magnetic field $\vec{B}^e = B_0 \vec{e}_y$ is perpendicular to channel's walls $y = \pm a$. If the width of the rectangular channel is much larger than its height, then the velocity of conducting fluid depends only on variable *y*, i.e $\vec{V} = V(y)\vec{e}_x$. We also suppose that flow rate of the fluid is kept constant, i.e. the average velocity of the fluid V_0 is constant:

$$V_0 = \frac{1}{2a}\int_{-a}^{+a} V(y)dy \ .$$

In the external magnetic field, the motion of the conducting fluid induces an electrical current of density $\vec{j} = \sigma\left[\vec{E} + \vec{V} \times \vec{B}^e\right]$. Due to a large conductivity of walls $z = \pm b$, one can suppose that $\vec{E} = E \cdot \vec{e}_z$, $E = const$ and since walls $y = \pm a$ are non-conducting, the total current flowing through the channel's cross section $z = const$, must be equal to zero. Therefore, induced electrical current density is $\vec{j} = \sigma B_0 \left[V(y) - V_0\right]\vec{e}_z$.

As a result of the interaction of the induced current with the magnetic field, the electromagnetic force $\vec{F}_{el} = \vec{j} \times \vec{B}^e = \sigma B_0^2 \left[V_0 - V(y)\right]\vec{e}_x$ influence on the fluid so that in the channel's central part, where $V(0) = \max V(y)$, the braking of the flow occurs, but near the channel's walls, where $V(\pm a) = 0$, the flow acceleration takes place. As a result, profile of velocities takes the shape, which is plane in the centre of the channel, but it sharply goes to zero near the channel's walls, instead of the parabolic distribution of velocity (so-called Pouozelle flow) occurring in the absence of the magnetic field. This flow is known as Hartman flow and this phenomenon is called Hartman effect.

In general, the system of MHD equations has the form:

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\frac{1}{\rho}\nabla p + \gamma \nabla^2 \vec{V} + \frac{\mu}{\rho}\left[rot\vec{H} \times \vec{H}\right], \qquad (1)$$

where $\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\vec{V}$ is the velocity of the fluid, p is the pressure, $\vec{H}$ is the intensity of magnetic field, σ is the conductivity, ρ is the density, γ is the kinematic viscosity and μ is the relative magnetic permeability of the fluid.

The dimensionless equations in inductionless approximation have the form:

$$\frac{L}{T\gamma}\frac{\partial \vec{V}}{\partial t} + \mathrm{Re}\left(\vec{V} \cdot \nabla\right)\vec{V} = -\nabla p + \nabla^2 \vec{V} + Ha^2\left[\vec{E} + \vec{V} \times \vec{B}^e\right], \qquad (2)$$

where $\mathrm{Re} = V_0 L / \gamma$ is the Reynolds number, $Ha = B_0 L\sqrt{\sigma / \rho\gamma}$ is the Hartmann number, $\vec{E}$ is the electrical field, B_0 is the intensity of the external magnetic field, L is the characteristic size, T is the characteristic unit of time.

New results, obtained in MHD domain, have a lot of practical applications, such as: control over motion of liquid metal in metallurgy; creation of new MHD pumps, which do not contain any movable elements and etc. Magnetohydrodynamics has also a very important application in astrophysics for the explanation of the nature of Earth's magnetic field presence.

Besides, MHD is useful in projecting reactor tokamak (self-cooled fusion liquid metal blanket for fusion reactor). It is proposed to use the liquid metal for the cooling of the reactor's walls and for a protection of solid surfaces against corrosion and breakdown. In equation (2) Hartman number $Ha = B_0 L\sqrt{\sigma / \rho\gamma}$ is a dimensionless parameter characterizing intensity of the magnetic field. In the cooling systems of thermonuclear reactor tokamak, the Hartman number is a number of orders $Ha = 10^3 - 10^4$. Therefore, it is very important to study MHD flows located in strong magnetic field.

One of the problems, which is planned to consider in this project is a problem on MHD flows in the entrance region of channel in strong magnetic field.

Studying MHD flows in a strong magnetic field, it is usually supposed that the surface of the channel's wall is ideally smooth. However, in a projected reactor tokamak, the value of the Hartman boundary layer in strong magnetic field becomes commensurable with the size of the roughness of the surface. Therefore, it is necessary to study the influence of the surface's roughness on MHD flow of a liquid metal. This problem also is planned to consider in this project.

2. Description of the Problem and Objectives

1) As a part of the PhD thesis of Elena Ligere [1], the problem on a plane jet in strong magnetic field, flowing into a plane channel through the split of finite width in the channel's lateral side, was studied. An exact analytical solution was obtained for the problems at Stokes and inductionless approximation by using integral transforms. In solving the MHD problem in Stokes approximation, the component $\left(\vec{V} \cdot \nabla\right)\vec{V}$ was neglected in equation (2). This component is taken partly into account in Oseen approximation. In this approximation one take $\left(\vec{V}_0 \cdot \nabla\right)\vec{V}$ instead of component $\left(\vec{V} \cdot \nabla\right)\vec{V}$ in (2), where V_0=const is, for example, a middling velocity of the fluid in the infinity. Oseen approximation gives the opportunity to obtain more exact solution of the problem. However, solving the problem on a flow of viscous fluid in the initial part of the channel by using the Oseen approximation, it is usually assumed that the velocity and the pressure of the fluid are given at the entrance of the channel (see.[4], [6]). In our opinion, these boundary conditions overdetermine the problem,

because for the uniqueness of the solution of the problem it is sufficient to give only the velocity at the entrance of the channel.

Therefore ***the first aim of project is:***

- Solve the similar problem by using Oseen approximation if the average velocity of the flow at the channel's cross-section in the infinity is constant and the velocity of the fluid is only given at the entrance of the channel. It is necessary to consider two cases: the magnetic field is parallel to the channel and the magnetic field is perpendicular to the channel's wall.
- Obtain an asymptotic solution [3] of the problem at large Hartmann number as $Ha \to \infty$.
- For the case, when magnetic field is parallel to the channel, to study the influence of Oseen approximation on the M-shaped profiles of the velocity.

2) The second problem, studied in Ligere's PhD thesis, is the problem on MHD flow arising in half-space due to the roughness of a plane surface located in strong magnetic field. Two-dimensional MHD flow was considered for two cases: if the roughness is located in a bounded region and if the roughness has the form of a doubly periodic function. The solution of the problem was obtained in linear approximation by integral transforms and it has the form of a double integral. Preliminary analysis shows that the obtained double integrals can be transformed to product of the two single integrals at large Hatrmann numbers, which can be evaluated analytically. This considerably simplifies numerical computations [2].

Therefore, ***the second aim of the project*** is:

- Transform obtained double integrals to product of the two single integrals.
- Obtain the asymptotic solution at Hartmann large numbers ($Ha \to \infty$) for the problem about two-dimensional MHD flow arising in consequence of the roughness of a plane surface.

3) ***The third aim of project*:**

Generalize problem 1 and problem 2 and solve the problems on the MHD flow in a plane channel, arising in consequence of the roughness of the channel's walls in strong magnetic field.

4) ***The forth aim:***

Two effective methods for determining integral characteristics in the problems on heat convection (for example, Rayleigh critical number) are the Bubnov - Galerkin and Ritz methods used in the calculus of variations [5]. Using only a few basic functions in the calculus of variations, one obtains that Rayleigh critical number differs from the exact one less than 1%. Since full pressure of the flow is also an integral characteristics, it must be perspective to use the calculus of variations for obtaining full pressure. The most important here will be selection of basic functions.

References

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