CTS FINAL REPORT
of Elena Ligere

Marie Curie Control Training Site Fellowship HPMT-GH-01-00278-136

CTS Host Institute: University of Aveiro, Portugal

Title of the Research Project: Analytical solutions to some magnetohydrodynamical control problems on a flow of conducting fluid

Home Supervisor: - Prof. Maximilian Ya. Antimirov (unexpectedly died during my CTS study)
- Associate Prof. Inta Volodko

CTS Host Supervisors: Prof. Delfim F. M. Torres

CTS stay: 4 months (15 March – 15 July 2005)

My Research project was devoted to solutions of some magnetohydrodynamical control problems on a flow of conducting in a strong magnetic field.

According to the first step of my CTS research, I have considered the following problem on MHD flow in a plane channel. The plane channel is located in region $-h \le \tilde{y} < h; -\infty < \tilde{x}, \tilde{z} < +\infty$. The fluid with prescribed velocity flows into the channel through a split in channel's wall $\tilde{y} = -\tilde{h}$ and flows out with the same velocity through the symmetrical split at $\tilde{y} = \tilde{h}$. Moreover, there exists the flow of the fluid with constant consumption that transforms into Poiseuille flow $\tilde{\vec{V}}_p(\tilde{y})$ far away from the split. I considered two cases: if the magnetic field is parallel to the channel and if the magnetic field is perpendicular to the channel's wall. The geometry of the flow for the longitudinal magnetic field is shown in Fig.1.

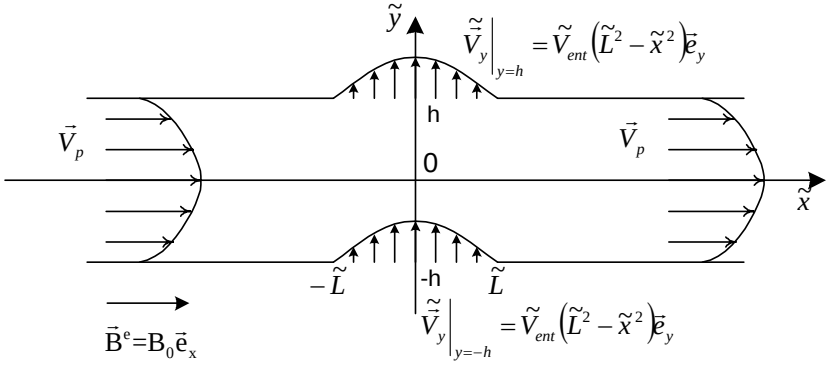

Fig.1. The geometry of the flow

As a part of my PhD thesis, the similar problems were solved in Stokes approximation. Oseen approximation gives the opportunity to obtain a more exact solution by partially taking into account the nonlinear component of the velocity in general MHD equations (in Stokes approximation this component is neglected). However, solving the problem on a flow of viscous fluid in the initial part of the channel by using the Oseen approximation, it is usually assumed that the velocity and the pressure of the fluid are given at the entrance of the channel. In my opinion, these boundary conditions overdetermine the problem, and for the uniqueness of the solution of the problem, it is sufficient to give only the velocity at the entrance of the channel. I solved the above mentioned problem in Ozeen and inductionless approximation if the only velocity of the fluid is given at the entrance of the channel. The analytical solution of the

problem was obtained by using Fourier transform in the Complex plane. The length of the initial part L_{init} is found. In this part the velocity $V_x(x, y)$ differs from the Poiseuille flow at less than 1%. Besides, an asymptotic solution of the problem is obtained at large Hartmann number as $Ha \to \infty$. The influence of Oseen approximation on the M-shaped profiles of the velocity was studied for the case, when magnetic field is parallel to the channel.

The second step of my CTS research was connected with the problem on two-dimensional MHD flow arising in a half-space due to the roughness of a plane nonconducting surface located in strong magnetic field. This problem partially was studied before my CTS fellowship training and the solution of the problem was obtained in the linear approximation and it has the form of a double integral. My aim was to transform obtained double integrals to product of two single integrals at large Hatrmann numbers. So that, each from these single integrals can be evaluated analytically. It considerably simplifies numerical computations. This transformation was successfully done. Numerical calculations of x- and y-component of velocity at large Hartmann number have been made. Moreover, streamlines of induced current was studied numerically.

As a third step of my research it was planned to study the influence of the roughness of the surface on the MHD flow of the conducting in a plane channel at the presence of a strong magnetic field. During my CTS training I only have begun the work with this problem. The solution of the problem isn't obtained yet. This problem needs more time.

During my CTS fellowship we (me and CTS fellow Ilona Dzenite, who also is at the University of Aveiro under Host Supervision of Prof. Delfim F. M. Torres) prepared preprint on integral transforms and its applications to some problems of nondestructive testing and magnetohydrodynamics:

> Antimirov M.Ya., Dzenite I.A. and Ligere E.S. ***Application of integral transforms to some problems of nondestructive testing and magnetohydrodynamics***. `Research report CM05/I-27, Department of Mathematics,` University of Aveiro, Portugal, June 2005, online version is available at http://www.pisharp.org/dspace/handle/2052/76

This preprint is based on content of the lectures which Prof. M.Ya. Antimirov was going to present at the Department of Mathematics in the University of Aveiro in Spring 2005. To our greatest regret, that couldn't happen because of unfortunate and sudden death of M.Ya.Antimirov. In honor to Prof. M.Antimirov, we did our best to finish and extend this preprint, as this is one of Prof. Antimirov's last works.

Recently, I began to consider some new problems of the Calculus of Variations for functionals with compositions. Together with Prof. Delfim Torres and his other CTS student, Ilona Dzenite, we are investigating the possible extension of Noether's theory for problems with compositions. This is very interesting and important for me because it is planed to use some methods of the Calculus of Variations in my PhD thesis for the solution of some problems of Mgnetohidrodynamical flow of conducting fluid in a channel.

I am very grateful to my CTS Host Supervisor Prof. Delfim F. M. Torres for value advice and consultations on numerical computations. All the talks I had with my CTS host supervisor Prof. Delfim Torres, was very important. I feel that the experience I received in the University of Aveiro was very important for my scientific growth.

I would like to thank CTS and University of Aveiro for the perfect working conditions and for the possibility to improve the work of my PhD thesis.

Date: 12.07.2005 CTS Fellow: CTS HOST Supervisor: