Actions Universitaires Intégrées Luso-Françaises

Final Report of the Research project *Application of Conservation Laws to Space Trajectography*

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1 Description of the Activities

1.1 Main Results

We have studied, in a unified way, the following questions related to the properties of Pontryagin extremals for optimal control problems with unrestricted controls: i) How the transformations, which define the equivalence of two problems, transform the extremals? ii) How to obtain quantities which are conserved along any extremal? iii) How to ensure that the set of extremals include the minimizers predicted by the existence theory? These questions are connected to: i) the Carathéodory method which establishes a correspondence between the minimizing curves of equivalent problems; ii) the interplay between the concept of invariance and the theory of optimality conditions in optimal control, which are the concern of the theorems of Noether; iii) regularity conditions for the minimizers and the work pioneered by Tonelli.

The dynamic optimization problems treated by the calculus of variations are usually solved with the help of the 2nd order Euler-Lagrange differential equations. These equations are, generally speaking, nonlinear, and very hard to solve. One way to address the problem is to obtain conservation laws of lower order than those of the corresponding Euler-Lagrange equations. While in Physics and Economics the question of existence of conservation laws is treated in a rather natural way, because the application itself suggest the conservation laws (e.g., conservation of energy, income/health law), from a strictly mathematical point of view, given a problem of the calculus of variations, it is not obvious how one might derive a conservation law or, for that matter, if it even has a conservation law. We developed computational facilities, based on a systematic method, which permits to identify functionals that have conservation laws. The central result used was the celebrated Noether's theorem. This theorem links conservation laws with the invariance properties of the problem (with symmetries), and provides an algorithm for finding conservation laws. Thus the problem is reduced to the one of finding the variational symmetries. We showed how a Computer Algebra System can help to find the symmetries and the conservation laws in the calculus of variations.

For nonsmooth Euler-Lagrange extremals, Noether's conservation laws cease to be valid. We proved that Emmy Noether's theorem of the calculus of variations is still valid in the wider class of Lipschitz functions, as long as one restrict the Euler-Lagrange extremals to those which satisfy the DuBois-Reymond necessary condition. In the smooth case all Euler-Lagrange extremals are DuBois-Reymond extremals, and the result gives a proper extension of the classical Noether's theorem. This is in contrast with the developments of Noether's symmetry theorems to the optimal control setting, which give rise to non-proper extensions when specified for the problems of the calculus of variations.

Particular attention has been done to the important relation between the invariance of an optimal control problem under a family of transformations, and the existence of preserved quantities along the Pontryagin extremals. Several extensions of Noether theorem were provided, in the direction which enlarges the

scope of its application. We have obtained a more general version of Noether's theorem for optimal control problems, which incorporates the possibility to consider a family of transformations depending on several parameters and, what is more important, to deal with quasi-invariant and not necessarily invariant optimal control problems. We trust that this latter extension provides new possibilities and we illustrate it with several examples, not covered by the previous known optimal control versions of Noether's theorem.

We have also obtained a generalization of Noether's invariance principle for optimal control problems with equality and inequality state-input constraints. The result relates the invariance properties of the problems with the existence of conserved quantities along the constrained Pontryagin extremals. A result of this kind was posed as an open question by Vladimir Tikhomirov, in 1986.

We introduced a new time-dependent definition of spline curves in $\mathbb{R}^n$, which extends a recent definition of vector-valued splines introduced by Rodrigues and Silva Leite for the time-independent case. Previous results are based on a variational approach, with lengthy arguments, which do not cover the non-autonomous situation. We proved that the previous results are a consequence of the Pontryagin maximum principle, and are easily generalized using the methods of optimal control. Main result obtained asserts that vector-valued splines are related to the Pontryagin extremals of a non-autonomous linear-quadratic optimal control problem.

The mathematical model of a real flexible elastic system with distributed and discrete parameters was studied. The problem is described by a partial differential equation with non-classical boundary conditions. Complexity of the boundary conditions results in the impossibility to find exact analytical solutions and, to address the problem, we investigated the use of the asymptotical method of small parameter together with the numerical method of normal fundamental systems of solutions. These methods allowed us to investigate vibrations, and a technique for determination of complex eigenvalues of the considered boundary value problem was developed. The conditions, at which vibration processes of different character take place, were defined. Dependence of the vibration frequencies on physical parameters of the hybrid system was studied. We showed that introduction of different feedbacks into the system allow one to control the frequency spectrum, in which excitation of vibrations is possible.

Newton's problem of the body of least resistance in a media of chaotically moving particles was solved in arbitrary dimension. The practical meaning of this problem is that in many real situations the temperature motion of particles cannot be neglected. The solutions were studied numerically for homogeneous monatomic rarefied ideal gases, in 2- and 3-dimensional cases. The optimal shapes were constructed, and bifurcation diagrams related to different types of solutions were obtained.

1.2 Cooperation in Teaching

Both Portuguese and French responsible for the project have visited SISSA, International School for Advanced Studies, Trieste, Italy, May 2004, to study the

possibility of a joint *Erasmus Mundus Master Course on Mathematical Control Theory* between the University of Aveiro, Portugal; the Université Paris-Sud, France; and SISSA, Italy. On late September we decided not to submit the project for Erasmus Mundus, because the University of Paris-Sud chose to support only one Erasmus Mundus project, in a different area than mathematics. However, our efforts were not in vain: in 2005 it is planned the exchange of some students between the University of Aveiro and the University of Paris-Sud, namely:

- at least one student from the "Master d'Ingénierie Mathématique d'Orsay", Univ. Orsay, should visit the University of Aveiro, under the responsibility of D. Torres, for the practical part of his master.
- From January, 23, to March, 23, 2005, Sofia Oliveira Lopes will visit the University of Orsay, under the responsibility of E. Trélat, with a CTS financial support, and with the recommendation of D. Torres.
- From September 2005, one of the students of D. Torres in the University of Aveiro, will visit the University of Orsay during three months, under the responsibility of E. Trélat, with a CTS financial support. A PhD is planned, to be commonly supervised by D. Torres and E. Trélat.

Recall that E. Trélat is responsible for a master in the University of Orsay, France, and that D. Torres is responsible for the "Control Theory Group" of the R&D unit "Centre for Research in Optimization and Control", which also dinamizes some courses of the master program on Mathematics of the Department of Mathematics of the University of Aveiro. Details of teaching of the French Master can be found on the web pages

`http://www.math.u-psud.fr/~lichnew/DESS/OptionAuto.html`
`http://www.math.u-psud.fr/~trelat/`

The goals of these options is to train and give advanced qualification to applied mathematicians and engineers in the areas of Optimization and Control, preparing students for a career in the research domain or in industries.

D. Torres and E. Trélat are still trying to develop an official way to achieve a common master between the University of Aveiro and the University of Orsay. For the time being, exchanges of students take place, and are still planned for the next future.

1.3 International Conferences Organized

Both Portuguese and French responsible for the project were involved in the organization of the *3rd Junior European Meeting on Control, Optimization, and Computation*, held at the University of Aveiro, 6–8 September 2004, Portugal.

1.4 Invited Sessions Organized

1. Optimal Control I, Optimization 2004, Lisbon, July 2004
2. Optimal Control II, Optimization 2004, Lisbon, July 2004
3. Mathematical Control Theory I, Controlo 2004, Faro, June 2004
4. Mathematical Control Theory II, Controlo 2004, Faro, June 2004
5. Degeneracy Phenomena in Optimal Control and Calculus of Variations, Controlo 2004, Faro, June 2004

2 List of Talks at International Conferences

1. E. Trélat, Contrôle optimal appliqué au problème de rentrée atmosphérique. CIFA 2004, Douz, Tunisie, November 22–24 2004.
2. Cristiana J. Silva and Delfim F. M. Torres, On the Classical Newton's Problem of Minimal Resistance, Third Junior European Meeting on "Control, Optimization and Computation", University of Aveiro, Portugal, September 6–8, 2004.
3. E. Trélat, Numerical methods in optimal control. Third Junior European Meeting on "Control, Optimization and Computation", University of Aveiro, Portugal, September 6–8, 2004.
4. E. Trélat, Numerical methods in optimal control, application to the problem of atmospheric reentry. Journée optimisation et systèmes non-linéaires, LAAS, Toulouse, may 2004.
5. E. Trélat, Méthodes numériques en contrôle optimal. Groupe de travail numérique, Univ. Orsay, march 2004.
6. Delfim F. M. Torres, Symmetry in the Calculus of Variations and Optimal Control, Young European Researchers in Mechanics Meeting 2004 (YERM2004), Instituto Superior Técnico, Lisbon, Portugal, January 2004.
7. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, 6th WSEAS International Conference on Applied Mathematics (Session on: Optimization and Applications), Corfu, Greece, August 17-19, 2004.
8. Delfim F. M. Torres, Lipschitzian Regularity of the Minimizing Trajectories in the Calculus of Variations and Optimal Control: a Survey, Third Junior European Meeting on "Control, Optimization and Computation", University of Aveiro, Portugal, September 6–8, 2004.

9. Paulo D. F. Gouveia and Delfim F. M. Torres, A Computer Algebra Package for Determining Symmetries and Conservation Laws in the Calculus of Variations, Invited Session: Optimal Control I, Optimization 2004, Lisbon, Portugal, 25 - 28 July 2004.

10. Paulo D. F. Gouveia e Delfim F. M. Torres, Computação Algébrica no Cálculo das Variações: Determinação de Simetrias e Leis de Conservação, Congresso Nacional de Matemática Aplicada e Computacional, XXVII CNMAC, (Brazilian Congress of Applied Mathematics and Computation), Porto Alegre, Brasil, 13-16, September 2004.

11. Rui C. Rodrigues and Delfim F. M. Torres, Linear-quadratic optimal control problems and spline functions in Euclidean spaces, First CTS Workshop, Coimbra, Portugal, July 2004.

3 List of Publications

3.1 Books

1. B. Bonnard, L. Faubourg, E. Trélat, Mécanique céleste et contrôle de systèmes spatiaux, to appear, Springer (2005), 283 pages.

2. E. Trélat, Contrôle optimal : théorie et applications, to appear, Vuibert (2005), 250 pages.

3.2 Articles in International Journals

3. Delfim F. M. Torres, Carathéodory-Equivalence, Noether Theorems, and Tonelli Full-Regularity in the Calculus of Variations and Optimal Control. Journal of Mathematical Sciences, Vol. 120, No. 1, 2004, pp. 1032-1050.

4. Delfim F. M. Torres, Quasi-Invariant Optimal Control Problems. PortugaliæMathematica, Vol. 61, Fasc. 1, 2004, pp. 97-114. [MR 2040245] [Zbl 1042.49015]

5. Delfim F. M. Torres, The Role of Symmetry in the Regularity Properties of Optimal Controls, Proceedings of Institute of Mathematics of National Academy of Sciences of Ukraine, Vol. 50, Part 3, pp. 1488-1495, 2004. [MR 2077966]

6. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, WSEAS Transactions on Mathematics, Issue 3, Vol. 3, 2004, pp. 620–624. [MR 2089373]

7. Delfim F. M. Torres, Proper Extensions of Noether's Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations, Communications on Pure and Applied Analysis, Vol. 3, No. 3, 2004, pp. 491–500. [MR 2098297]

3.3 Accepted Papers

8. Delfim F. M. Torres, A Noether Theorem on Unimprovable Conservation Laws for Vector-Valued Optimization Problems in Control Theory, Accepted (paper ID: 131) in the Conference Proceedings of ICMSAO'05 – First International Conference on Modeling, Simulation and Applied Optimization, February 1-3 2005, American University of Sharjah, United Arab Emirates, 2005.

9. E. Trélat, Contrôle optimal appliqué au problème de rentrée atmosphérique, to appear, CIFA 2004.

10. Delfim F. M. Torres and Alexander Yu. Plakhov, Optimal Control of Newton-Type Problems of Minimal Resistance. Accepted to Rediconti del Seminario Matematico dell'Università e del Politecnico di Torino.

11. B. Bonnard, L. Faubourg, E. Trélat, Optimal control of the atmospheric arc of a space shuttle and numerical simulations by multiple-shooting techniques, to appear in Math. Models Methods Applied Sci. **2**, 15 (2005).

12. Alexander Yu. Plakhov and Delfim F. M. Torres, Newton's aerodynamic problem in a medium of chaotically moving particles. Accepted to Sbornik: Mathematics.

13. Rui C. Rodrigues and Delfim F. M. Torres, Generalized splines in $\mathbb{R}^n$ and optimal control. Accepted to Rend. Sem. Mat. Univ. Pol. Torino.

3.4 Research Reports (Preprints)

14. Paulo D. F. Gouveia and Delfim F. M. Torres, Computação Algébrica no Cálculo das Variações: Determinação de Simetrias e Leis de Conservação, Cadernos de Matemática CM04/I-23, Dep. Matemática, Univ. Aveiro, September 2004.

15. Olena V. Mul and Delfim F. M. Torres, Analysis of Vibrations in Large Flexible Hybrid Systems, Cadernos de Matemática CM04/I-30, Dep. Matemática, Univ. Aveiro, December 2004.