**POST-DOC WORK PROGRAM for Moulay Rachid Sidi Ammi,**
**to be carried out at the Department of Mathematics, University of Aveiro, Portugal**
**with FCT fellowship SFRH/BPD/20934/2004**

*Title of the proposed investigations:* **Thermistor Problems and Optimal Control**

*Start date:* **September 2005**

*Duration:* **12 months**

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## Abstract

Preliminary work shows the contributions that control theory can bring to the electric heating (see for example [Masserey 2003]). Although control simulation algorithms permit to carry out the heating tests with the advantage of removing the costs associated with the realization of prototypes, to determine the best control parameters, like the supply voltage, the frequency, or the heating time, is still an open question. In this work we propose to address the problem by application of optimal control techniques. We claim that optimization algorithms of optimal control make possible to achieve the wished goal in an effective way.

## State of the art

Nowadays the technique of electric heating, which is based on the generation of electrical currents in a conducting part, is widespread in industry. The development of such devices, by experimental trial runs, can prove very expensive for the improvement of a process and the comprehension of the physical phenomena intervening during the heating. One puts then the question of optimizing the heating. In other words, we would like to know the parameters, such as the necessary tension and frequency, in order to obtain a controlled heating. During the last decade, many research articles and PhD theses were written on the subject. Here we just mention the thesis of [Paieitti 1998] and [Masserey 2003]; and papers by [Touzani 1996] and [Rappaz & Swierkosz 1996]. The main work was to find a good mathematical model and objective function to characterize the heating. In order to implement a technique of optimization, it is necessary to calculate the gradient of the characteristic function, according to the control parameters. Our goal consists

in seeking an optimal solution of the heating problem, respecting the constraints imposed by the problem while exploiting the control parameters,

## Objectives

Our objective is to extend current research on thermistor problems by application of optimal control theory to the electric heating. Mathematical tools necessary for the analysis of the problem include:
a) Methods of optimization.
b) Theory of control and optimal control.
c) Numerical analysis.
d) Functional analysis.
It is important to point out that this postdoctoral research project is interdisciplinary by nature, and original research in collaboration with members of the cotg-CEOC, working at the Department of Mathematics of the University of Aveiro, is planned. In particular, we mention the possible synergies with the Research Project "Impedance Change and the Calculus of Variations", which is being carried out at the University of Aveiro under the scientific supervision of Prof. Delfim F. M. Torres and the support of the European Commission Marie Curie Control Training Site.

## Detailed descriptions

In recent years my research has been concerned with mathematical analysis of the so-called thermistor problem which arise, for example, in the electric heating of a conductor. It is a very substantial long-term project, which on the one hand is of great practical importance, and on the other hand involves deep and highly nontrivial mathematics. The proposed post-doc consists in the continuation of the current project, benefiting from the expertise on optimal control of my Scientific Coordinator, Prof. Delfim F. M. Torres, and his research group in Aveiro, Portugal. Precisely, we are interested in the optimal control of the electric heating process of a conductor. This problem has been posed by several industries with the framework of an industrial process of manufacture of aluminium parts, called thixoforming. The quality of the part obtained depends directly on the quality of the heating. It is thus significant to be able to optimize the control parameters (supply voltage, the frequency, the time of heating, ...) of the electric heating in order to achieve this goal. Other parameters can also be taken into account to guarantee the quality of the heating.
The idea is to use numerical analysis techniques to solve the optimal control

problem. The whole proceeding phenomena, during the heating, are described by a nonlinear partial derivative equations with quadratical growth in the gradient. From physical considerations and following works of A. Masserey, we build an objective function J characteristic of the control which one wants to carry out. By a perturbation method, we plan to calculate the gradient of the cost function following the control parameters. This technique brings us to the calculations of two associated adjoint problems, one of the electromagnetism, the other for thermics. These problems, being solved, will make possible to evaluate the gradient of the objective function. The value of control variables, the gradient, and the function J, can be transmitted to a descent algorithm based on a quasi-Newton method, which gives us the new set of parameters. Then, we plan to reiterate the process until satisfying a stop criterion, which must depend on the required precision. Finally, we plan to obtain the optimal parameters. For solving this problem, we may use a combination of a finite difference method for the time discretization, and of a finite elements method for the space discretization. We shall present numerical tests and optimization results. We shall also examine the performance of our method for the optimal control of electric heating.

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