Control Theory Group (cotg)
Research 2003-2005

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1 Identification

Title: Control Theory Group

Short Title: cotg

Main scientific area: Mathematics

Principal investigator: Delfim F. M. Torres

Duration in months: 36

Starting date: 01-01-2003

Research Team:

1. Delfim F. M. Torres (PhD, Coordinator, 30%)
2. Eugénio M. Rocha (PhD, 30%)
3. Manuel Guerra (PhD, 30%)
4. Paulo D. F. Gouveia (MSc, PhD student, 40%)

2 Technical Addendum

Summary (max. 200 words)

The purpose of mathematical control theory is to analyze the properties of controlled systems, i.e., dynamic systems in which one can act using a command or control. Controls can be introduced in most types of systems, like systems of

differential or difference equations. Control systems appear naturally in many areas, such as mechanics, biology and economics. The goal can be, for example, to steer the system from an initial state to a prescribed target, according to certain constraints on the state and/or the control, or to design trajectories that minimize some given cost (optimal control). Since the 1950's, with the discovery of Pontryagin's Maximum Principle, control theory had a great development and became a recognized field of mathematical research. Optimal control can be viewed as a generalization of the calculus of variations.

The aim of "cotg" is to make advances in the following areas:

1. Nonlinear Control Theory and Optimal Control.

2. Calculus of Variations.

3. Motivated Applications and Computational Mathematics.

Objectives (max. 200 words)

1. To obtain new versions of Pontryagin's maximum principle covering classes of generalized controls; to give geometric characterizations of generalized extremals and accessible sets; to investigate singular optimal control problems; to obtain explicit formulas for the Chen logarithm and develop a formal control theory with applications to stabilizability and noncommutative symmetric functions; to obtain conditions for boundedness of optimal controls; to investigate the role of symmetry and conservation laws.

2. Extension of Noether's theorem of the calculus of variations to the wider class of Lipschitz functions, and obtain a corresponding formulation for higher-order variational problems; to obtain conserved quantities along all minimizers of the problems; to identify new and more general classes of well and bad behaved problems in the calculus of variations.

3. To study systems of rolling bodies by designing time-variant feedback stabilizers with possible application to robotic manipulators; to develop a computer algebra package for determining symmetries and conservation laws in the calculus of variations and optimal control; to prove convergence of digital arithmetic algorithms by using results on feedback stabilization; develop numerical and asymptotical methods to study nonlinear dynamical control systems with distributed and discrete parameters, decreasing the effect of vibrations.

Relationship with the state of the art (max. 500 words)

1. Singular linear-quadratic problems with finite lower bound always admit a generalized optimal control of an appropriate class. Generalized solutions for these problems admit a simple geometric characterization using

"generalized Hamiltonian flows" (Guerra'00'01). Different methods exist to construct classes of generalized controls for other types of problems (Miller&Rubinovich'01,Sarychev'91). In general, existence of generalized Hamiltonian flows seems to be related to existence of local feedback linearizations, described in (Jakubczyk'98'01).

Formulas for the Chen logarithm of a nonlinear control system are quite useful as its alternative, coordinates of second kind (Sussmann'86), in resolving problems of controllability (Sussmann'87, Murray et al'94), stabilizability (Rocha&Sarychev'02), motion planning (Hermes'89), or averaging of mechanical systems (Bullo'02). Several results exists in control theory (Kawski'00) or related with the continuous Baker-Campbell-Haudorff formula in quantum mechanics (Vinokurov'69), but none presented explicit formulas in a Hall basis, turning them useless for general applications.

The fundamental ideas of regularity in optimal control, for linear time-invariant control systems, are due to Clarke&Vinter'90. Results for the control-affine case were obtained by Sarychev&Torres'00. We intend to generalize such results to nonlinear problems.

In the optimal control setting, the relation between invariance of a problem and the existence of expressions which are constant along any of its extremals, has been obtained in the publications by van-der-Schaft'82 and Sussmann'95. Recent extensions were obtained by Torres'01'02. We will attempt now to enlarge the range of application of the previous results by extending the very concept of invariance.

2. It is well-known that problems in the calculus of variations may present absolutely continuous solutions which are not Lipschitz, and which fail to satisfy the Euler-Lagrange equations (Ball&Mizel'85, Sarychev&Torres'00). Different but related pathologies are also possible (Sarychev'97). We want to investigate a new perspective on the results concerning the Lipschitz regularity of the minimizing trajectories in the calculus of variations, by using appropriate conservation laws (Torres'02). For that it is important to obtain a proper extension of the classical Noether's symmetry theorem (Heinricher&Mizel'88).

3. The problem of controlling rolling bodies inspired much attention in the literature due to its importance in robotic applications. An intrinsic geometric method for two solid bodies rolling over each other, without slipping and twisting, was proposed by Agrachev&Sachkov'99.

Many powerful and versatile Computer Algebra Systems are available nowadays, putting at our disposal sophisticated environments of mathematical computing. Symbolic methods for control is now a very active research area. In particular, we are interested to develop a package to find the symmetries and the conservation laws in the calculus of variations and optimal control.

Digital Arithmetic has been an active field of research over the past two decades. A large number of algorithms have been published for hardware implementation. However, a solid mathematical framework is still missing.

Analysis and control of oscillations in dynamical systems with distributed and discrete parameters were investigated by Kravchenko&Mul&Shut'99. Such systems are widespread in heavy, extractive and manufacturing industry, and also in space-system engineering.

3 Tasks

Nonlinear Control Theory and Optimal Control

Research team: (Total = 55% per month)

1. Delfim F. M. Torres (10%)
2. Eugénio M. Rocha (15%)
3. Manuel Guerra (30%)

Task duration – months: 36

Persons*month: 19.8 (0.55 * 36)

Expected results (max. 100 words): New versions of Pontryagin's maximum principle or other "extremality conditions", covering various classes of generalized controls; geometric description of generalized extremals and accessible sets; explicit formulas for the Chen logarithm, and a formal control theory; new regularity conditions for boundedness of optimal controls; Lipschitzian regularity of optimal trajectories; extensions of Noether's first and second theorems, in the direction which enlarges the scope of its application, and corresponding discrete-time analogous; useful necessary and sufficient conditions of invariance.

Task description (max. 200 words): We plan to generalize previous results (Guerra'00'01) to affine and nonautonomous systems. We consider commutative and noncommutative cases. We expect to construct generalized solutions having good properties and obtain extremality conditions using the appropriate generalized Hamiltonian flow. Short segments of generalized extremals have a special structure in linearized local coordinates. We expect to use this relationship to provide global geometric descriptions of generalized extremals, even when global linearization does not exist.

We plan to obtain expressions for the logarithm of the Chen-Fliess series of nonlinear control systems, as a development of high-order averaging. The idea is to propose a calculus to deal with formal non-autonomous ordinary differential equations evolving on an algebra of formal series.

We will address the question: under what conditions one can assure optimal controls to be bounded? This question is related to the one of Lipschitzian regularity of optimal trajectories, and the answer to it is crucial for closing the gap between the conditions arising in the existence theory and necessary optimality conditions. We will look for conditions beyond standard hypotheses of the existence theory.

We will explore the relation between Noether's theorem, symmetry, Pontryagin's maximum principle, invariance under a family of transformations, and the existence of preserved quantities.

Calculus of Variations

Research team: (Total = 30% per month)

1. Delfim F. M. Torres (10%)
2. Paulo D. F. Gouveia (20%)

Task duration – months: 36

Persons*month: 10.8 (0.30 * 36)

Expected results (max. 100 words): A proper extension of Noether's theorem of the calculus of variations to the wider class of Lipschitz functions; new invariants for problems of the calculus of variations with higher-order derivatives; conserved quantities along all the minimizers of the fundamental problem of the calculus of variations, even for those absolutely continuous minimizers which does not satisfy the classical Euler-Lagrange equations; new Lipschitzian regularity conditions for the problems of the calculus of variations; more general classes of bad behaved problems.

Task description (max. 200 words): A difficulty which appears in the calculus of variations (and optimal control) is to prove that the minimizers predicted by the existence theory satisfy the corresponding Euler-Lagrange equations. This is due to the fact that existence of minimizers is given in the class of absolutely continuous functions, while Euler-Lagrange equations require more regularity from the minimizers in order to be obtained. We propose a new perspective on the results concerning the regularity of the minimizing trajectories. Our claim is that such results are fruit of the existence of certain symmetry properties, and follow from appropriate conservation laws. For nonsmooth extremals, Noether's conservation laws cease to be valid. We expect to obtain new results showing that Noether's theorem is still valid in the wider class of Lipschitz functions, as long as some appropriate restriction of the Euler-Lagrange extremals is taken in account. We intend to obtain a proper extension, in contrast with the recent developments of Noether's symmetry theorems to the optimal control setting, which give rise to non-proper extensions when specified for the problems of the calculus of variations.

Motivated Applications & Computational Mathematics

Research team: (Total = 45% per month)

1. Delfim F. M. Torres (10%)
2. Eugénio M. Rocha (15%)
3. Paulo D. F. Gouveia (20%)

Task duration – months: 36

Persons*month: 16.2 (0.45 * 36)

Expected results (max. 100 words): Time-variant feedback stabilizers for systems of rolling bodies without twisting or slipping; results which turn the origin of rolling bodies control systems into locally asymptotically stable equilibrium points; a computer algebra package for determining symmetries and conservation laws automatically, from the data of the problems of the calculus of variations or optimal control; convergence results of digital arithmetic algorithms; numerical and asymptotical methods which decrease the effect of vibrations for nonlinear dynamical control systems with distributed and discrete parameters.

Task description (max. 200 words): We will study the stability and asymptotic stability of time-variant differential control equations, trying to design time-variant stabilizers for nonlinear control systems, in particular for nonholonomic systems of two bodies rolling one over the other without twisting or slipping. Our approach is to use Chen logarithm expressions, the monodromy map of a flow, and Lyapunov functions. Some computational tools will be developed.

The equations given by necessary optimality conditions are, generally speaking, nonlinear, and very hard to solve. One way to address the problem is to obtain conservation laws to lower the order of the equations. The crucial point is to derive the invariance-variational-symmetries. We will develop symbolic computational facilities, based on necessary and sufficient invariance-conditions, to identify symmetries and conservation laws.

We intend to model redundant arithmetic algorithms as discrete-time time-variant control systems, with the strings of digits being treated as integer-valued controls, and obtain convergence of digital arithmetic algorithms by using results on feedback stabilization.

We will study the effect of vibration on stability of mechanical systems. We will be particularly interested in nonlinear control systems with distributed and discrete parameters. Both numerical and asymptotical methods will be developed.

4 Scientific Report 2003

Publications

Books

1. A. Gugushvili, O. Khutsishvili, V. Sesadze, G. Dalakishvili, N. Mchedlishvili, T. Khutsishvili, V. Kekenadze, and Delfim F. M. Torres, Symmetries and Conservation Laws in Optimal Control Systems (book in Georgian), Georgian Technical University, Tbilisi, 2003. (ISBN 99940-14-53-6)

Chapters in Books

1. Delfim F. M. Torres, A Proper Extension of Noethers Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Lagrangian and Hamiltonian Methods for Nonlinear Control. Editors: Astolfi, Gordillo and van der Schaft, 2003, pp. 195-198.
2. Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems. Control Applications of Optimisation. Editors: Bars and Gyurkovics, 2003, pp. 33-38.
3. Eugénio M. Rocha, "On computation of the logarithm of the Chen-Fliess series for nonlinear systems", in Zinober, Alan (ed.) et al., Nonlinear and adaptive control. Lect. Notes Control Inf. Sci. 281, 317-326, 2003
4. Manuel Guerra, On nonautonomous singular L-Q problems, Lagrangian And Hamiltonian Methods In Nonlinear Control (Editors: A. Astolfi, F. Gordillo, A. van der Schaft) pp. 189-194, 2003

Articles in International Journals

1. Delfim F. M. Torres, Gauge Symmetries and Noether Currents in Optimal Control, Applied Mathematics E-Notes, Vol. 3, 2003, pp. 49-57
2. Delfim F. M. Torres, Lipschitzian Regularity of the Minimizing Trajectories for Nonlinear Optimal Control Problems, Mathematics of Control, Signals, and Systems (MCSS), 16, 2003, pp. 158-174.

Proceedings with Referee

1. Delfim F. M. Torres, A Proper Extension of Noethers Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Proceedings of the International Federation of Automatic Control, 2nd Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control (invited session on "Optimal Control"), Seville, Spain, April 3-5, 2003, pp. 225-228.

2. Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems. Proceedings of the International Federation of Automatic Control, Workshop on Control Applications of Optimization - CAO2003, Visegrád, Hungary, 30 June - 2 July 2003, pp. 44-49.
3. Manuel Guerra, On nonautonomous singular L-Q problems. Second Workshop on Lagrangean and Hamiltonian methods for nonlinear control, April, 3-5 2003, Seville, Spain. Astolfi, A.; van der Schaft, A.J.; Gordillo, F. eds. Preprints of the IFAC, 2003 (CD-ROM), pp. 219-224.

Talks at International Conferences

1. Delfim F. M. Torres, A Proper Extension of Noether's Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations, Invited Session on Optimal Control, 2nd IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Control, Sevilla, Spain, April 2003
2. Delfim F. M. Torres, The role of symmetry in the regularity properties of optimal controls, Symmetry in Nonlinear Mathematical Physics, Kiev, Ukraine, June 2003
3. Delfim F. M. Torres, Integrals of Motion for Discrete-Time Optimal Control Problems, session "Optimal Control" (co-chair), IFAC Workshop on Control Applications of Optimization, CAO 2003, Visegrád, Hungary, July 2003
4. Delfim F. M. Torres, The Noether Principle of Optimal Control, Session on Optimal Control Theory, Eight Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics: Theory and Applications in Economics and OR/MS, Vienna, Austria, May 2003
5. Eugénio M. Rocha, Computation of coordinates of first kind for nonlinear control systems, SciCADE03, Norway, Jul/2003
6. Manuel Guerra, On nonautonomous singular L-Q problems. Second Workshop on Lagrangean and Hamiltonian methods for nonlinear control, April, 3-5 2003, Seville, Spain.
7. Manuel Guerra, Discontinuous Hamiltonian flows for nonlinear control systems. Second Junior European Meeting "Control Theory and Stabilization", Politecnico de Torino, Torino, Italia, Dezembro de 2003

5 Scientific indicators of 2004 (January-June 2004)

Publications

Accepted Papers

1. Delfim F. M. Torres, Proper Extensions of Noether's Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations. Accepted for publication in the journal Communications on Pure and Applied Analysis.
2. Rui C. Rodrigues and Delfim F. M. Torres, Generalized splines in $\mathbb{R}^n$ and optimal control. Accepted to Rend. Sem. Mat. Univ. Pol. Torino.
3. Manuel Guerra, Discontinuous Hamiltonian flows for nonlinear control systems, Rend. Sem. Mat. Univ. Pol. Torino, accepted March 2004.

Articles in International Journals

1. Delfim F. M. Torres, Carathéodory-Equivalence, Noether Theorems, and Tonelli Full-Regularity in the Calculus of Variations and Optimal Control. Journal of Mathematical Sciences, Vol. 120, No. 1, 2004, pp. 1032-1050.
2. Delfim F. M. Torres, Quasi-Invariant Optimal Control Problems. Portugaliæ Mathematica, Vol. 61, Fasc. 1, 2004, pp. 97-114.
3. Delfim F. M. Torres, The Role of Symmetry in the Regularity Properties of Optimal Controls, Proceedings of Institute of Mathematics of National Academy of Sciences of Ukraine, Vol. 50, Part 3, pp. 1488-1495, 2004.
4. Paulo D. F. Gouveia and Delfim F. M. Torres, Smarandache Sequences: Explorations and Discoveries with a Computer Algebra System, Smarandache Notions Journal, Vol. 14, 2004, pp. 5-22.
5. Eugénio Rocha, A.Sarychev, A.Pereira, R.Rodrigues. Control-theoretic methods for design of algorithms of digital arithmetic. Journal of Mathematical Sciences, vol.120 (I), 2004.
6. Manuel Guerra, Distribution-like Hamiltonian flows and generalized optimal controls. J. Math. Sci., Vol. 120 No 1, 2004, pp 895-918.

Articles in National Journals

1. Delfim F. M. Torres, Números Felizes e Sucessões Associadas: Digressões com o Maple, Educação e Matemática no77, Revista da Associação de Professores de Matemática, Março/Abril de 2004, pp. 35–38.

Research Reports

1. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, Cadernos de Matemática CM04/I-12, Dep. Matemática, Univ. Aveiro, 2004.

PhD Thesis

1. Eugénio M. Rocha, An Algebraic Approach to Nonlinear Control, PhD Thesis, Univ. of Aveiro, 2004.

Talks at International Conferences

1. Delfim F. M. Torres, Symmetry in the Calculus of Variations and Optimal Control, Young European Researchers in Mechanics Meeting 2004 (YERM2004), Instituto Superior Técnico, Lisbon, Portugal, January 2004

2. Manuel Guerra, Discontinuous Hamiltonian flows and generalized extremals for nonlinear control systems. Workshop "Geometry of Control Systems and Distributions". Banach Center, Polish Academy of Science, Warsaw, 17 de Maio de 2004.