Actions Universitaires Intégrées Luso-Françaises
Research project:
Application of Conservation Laws to Space Trajectography

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Persons responsible for the project:

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Title : Application of conservation laws to space trajectography.

Aim : The aim is to initialize a collaboration between the University of Aveiro, Portugal, and the University of Orsay, France, in view of developing:

- a research project in space trajectography;
- collaborative teaching and research between the master programs on control theory of the University of Aveiro and the University Paris XI.

We hope to strength the research team with students from the Master's degree of both countries, and to motivate some of the students to continue with PhD studies in the area of the project.

Conservation laws and space trajectography.

The state of the art. One of the main tasks in space trajectography is to analyze and determine numerically the accessible set associated to a control problem. This study is motivated by the following facts. The presence of constraints, for instance on the controls, or technological constraints, makes the problems of trajectography difficult, which has several consequences:

1. problem of existence of admissible trajectories: it is in general difficult to know if it is possible, starting from a point or a given set, to reach a target *without violating the constraints*,

2. problem of generating accessible points: the aim is to develop methods which would determine *all* accessible targets, with respect to the constraints,

3. numerical problems of optimal control.

The existence of solutions satisfying the constraints is a difficult matter. The idea is to give a tool in view on improving the convergence of numerical algorithms, for instance in optimal control, but also to determine all accessible targets from a given initial set.

This kind of problem happens at least in two cases:

1. the problem of atmospheric re-entry,

2. the problem of orbit transfer with ionic motors.

In the first case, it is important to control these aerocapture techniques in order to apply them to problems of guidance of aeroassisted orbital transfers, to the development of reprocessable satellite launchers (this is an important financial stake), or to problems of re-entry in the atmosphere: this is the subject of the famous project *Mars Sample Return* developed by the CNES (French Space Agency). One of the authors of the present project has long been working on that subject, see [3, 4, 5, 18]. This is the continuation of a collaboration between the CNES, the Ecole des Mines of Paris, and the university Paris XI.

In the second case (orbital transfer), the presence of many constraints makes the problem very hard, and the question of the existence of admissible trajectories is mainly posed. The use of ionic motors implies that the control is very limited and constrained. From a theoretical point of view the situation has first to be analyzed on a period. Numerically one has then to generate admissible trajectories for this problem. This matter has been already partially investigated by researchers from the university Paris XI and the CNES, see [6].

Research goals. The approach we propose is based on *conservation laws*. Indeed a very complete theory was developed by one of the authors in a series of articles [11, 12, 13, 14, 15, 16, 17], and consists in exhibiting conserved quantities along the extremals of a given control problem. The author developed very efficient tools and constructive methods to obtain such conserved quantities. Typical applications are to lower the order of the underlying differential control system, and this simplifies the solution to an optimal control problem. This also permits to prove the existence of minimizers.

Our hope is to be able to give new conservation laws at least in both nontrivial problems cited above. In particular in the problem of orbital transfer trajectories are quasi-periodic, and thus the knowledge of some conservation law is crucial in order to numerically approximate them.

Another approach consists in introducing an optimization criterion so that the accessible set is computed using the level sets of the value function associated to the problem. The classical theory states that this function is solution of the

Hamilton-Jacobi-Bellman equation. As proved in [7, Ch. 13], [10, 2], the use of conservations laws permits to simplify this equation, by lowering the dimension of the space. But recently many efficient numerical methods were developed in order to solve the Hamilton-Jacobi equation (see [1, 8]) by computing the level sets of the value function. They are very efficient in low dimension.

These techniques thus permit to compute directly the accessible set for our problem. The algorithmic complexity is *linear* in function of the number of points of discretization. They were implemented in a very efficient way in problems of dimension 3 and sometimes more.

Using the procedure of lowering dimension explained above, we hope to be able to apply these methods to space trajectography. The final task is to develop algorithms capable to give a precise approximation of the accessible set.

Cooperation in teaching and research between the master courses on control theory of Aveiro and Orsay One of the authors is responsible for a master (still called, in French, "DESS") in the University of Orsay, France; while the other is responsible for the course "Mathematical Theory of Nonlinear Control" of the master program on Mathematics, option "Optimization and Control Theory", of the Department of Mathematics of the University of Aveiro. Details of teaching of the French Master can be found on the web pages

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http://www.math.u-psud.fr/~lichnew/DESS/OptionAuto.html
http://www.math.u-psud.fr/~trelat/
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while information on the Portuguese Master degree and teaching are found at

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http://www.mat.ua.pt/ensino/mestrados/OTC
http://www.mat.ua.pt/delfim
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The option called "Automatic" in the French program prepares students to become Engineers in industries applying control theory. For the moment all courses are located in the Univeristy of Orsay, and are organized by the Math. Laboratory. The industries concerned by this formation are numerous: aerospace, automobile, robotics, aeronautics, internet, chemist, ...

The goals of the option "Optimization and Control Theory" of the Portuguese MSc program at Aveiro, is to train and give advanced qualification to Applied Mathematicians and Engineers in the areas of Optimization and Control, preparing students for a research career. Our objective is to develop a collaboration between the University of Aveiro, in Portugal, and the University of Orsay, in France. Some lectures of the "Mathematical Theory of Nonlinear Control" course of the Portuguese author will be given by the French author; and, similarly, some of the lectures in France will be given to the French students by the Portuguese member. MSc Projects will be presented to the students in special seminars (these seminars are part of the Portuguese MSc degree curriculum) and interested students can take part of the thesis research in the other country.

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