

Scientific Report 2004

Control Theory Group (cotg)

Contents

1	Identification, Researchers, and Students	2
2	Description of the Activities during 2004	3
2.1	Main Results	3
2.2	Visiting Appointments of cotg members	5
2.3	Invited Researchers	6
2.4	Participation in Projects	6
3	Publications during 2004	6
3.1	Books	6
3.2	Articles in International Journals	6
3.3	Chapters in Books	7
3.4	Articles in Nacional Journals	7
3.5	Proceedings with Referee	7
3.6	Research Reports (Preprints)	8
3.7	PhD and MSc Thesis	8
3.8	Accepted Papers	9
4	Talks during 2004	9
4.1	Talks at International Conferences	9
4.2	Talks at National Conferences	10
4.3	Seminars	10
5	Organization of Conferences during 2004	11
6	Organization of Special/Invited Sessions during 2004	11
7	Scientific Indicators of 2004	12

1 Identification, Researchers, and Students

Title of the Research Group: Control Theory Group

Short Title: cotg

Main scientific area: Mathematics

Principal investigator: Delfim F. M. Torres

Research Team:

1. Delfim F. M. Torres (PhD, Coordinator, 30%)
2. Eugénio M. Rocha (PhD, 30%)
3. Manuel Guerra (PhD, 30%)
4. Paulo D. F. Gouveia (MSc, PhD student, 40%)
(Supervisor: Delfim F. M. Torres)

Post-doc:

5. Olena V. Mul (PhD, 100% post-doc since July 2004)
Associate Professor of Physics and Mathematics
Ternopil Academy of National Economy, Ukraine
(Scientific Coordinator: Delfim F. M. Torres)

MSc Students:

6. Cristiana Silva (Supervisor: Delfim F. M. Torres)
7. Joana Costa (Supervisor: Delfim F. M. Torres)
8. Luísa Marques (Supervisor: Delfim F. M. Torres)

Scientific Cooperation:

- Maximilian Ya. Antimirov, Riga Technical University, Latvia
- Zbigniew Bartosiewicz, Bialystok Technical University, Poland
- Ugo Boscin, SISSA, Trieste, Italy
- Yacine Chitour, Université Paris-Sud XI, Orsay, France
- Matthias Kowski, Arizona University, USA
- Boris Miller, Russian Academy of Sciences, Russia
- Valida Sesadze, Georgian Technical University, Tbilisi, Georgia
- Emmanuel Trélat, Université Paris-Sud XI, Orsay, France

2 Description of the Activities during 2004

2.1 Main Results

Progress has been made in the characterization of generalized extremals and accessible sets for nonlinear control systems. Some aspects of the global synthesis for nonlinear systems affine with respect to control where improved.

Research is being actively carried into the links between local feedback linearizability of control systems and the structure of global extremal synthesis. We are confident that in a very near future, some of the results already obtained will be extended into classes of affine systems with high order of singularity and/or noncommutative affine systems.

At the same time, research is under progress, in collaboration with Boris Miller (Russian Academy of Science), with the aim of characterizing the accessible set of affine control systems with L_1 -bounded controls without commutativity assumptions.

These two research problems, are closely related but use different sets of theoretical tools. To understand the relationship and the possibilities of interplay between the theoretical tools used in these two lines of research is a long-term goal.

Research is recently started with the aim of characterizing sequences of controls whose corresponding trajectories approximate integral curves of high-order Lie brackets in smooth control system of type $\dot{x} = \sum f_i(x) u_i$. Here we follow a suggestion made by Andrei Agrachev.

A mathematical framework, which permits to treat redundant normalization algorithms in Digital Arithmetic as discrete-time time-variant dynamical control systems, with the strings of digits being treated as integer-valued controls, was developed. Results on time-variant feedback stabilization for control systems were used to obtain a global proof of convergence of the algorithms for the elementary functions.

A minimal form for the Chen logarithm was obtained on a Hall basis, improving previous results. By connecting nonlinear control theory with noncommutative symmetric functions, minimal Lie-relations relating different classes of power sums symmetric functions, defined by Gelfand, were obtained.

We have studied, in a unified way, the following questions related to the properties of Pontryagin extremals for optimal control problems with unrestricted controls: i) How the transformations, which define the equivalence of two problems, transform the extremals? ii) How to obtain quantities which are conserved along any extremal? iii) How to assure that the set of extremals include the

minimizers predicted by the existence theory? These questions are connected to: i) the Carathéodory method which establishes a correspondence between the minimizing curves of equivalent problems; ii) the interplay between the concept of invariance and the theory of optimality conditions in optimal control, which are the concern of the theorems of Noether; iii) regularity conditions for the minimizers and the work pioneered by Tonelli.

The dynamic optimization problems treated by the calculus of variations are usually solved with the help of the 2nd order Euler-Lagrange differential equations. These equations are, generally speaking, nonlinear, and very hard to solve. One way to address the problem is to obtain conservation laws of lower order than those of the corresponding Euler-Lagrange equations. While in Physics and Economics the question of existence of conservation laws is treated in a rather natural way, because the application itself suggest the conservation laws (e.g., conservation of energy, income/health law), from a strictly mathematical point of view, given a problem of the calculus of variations, it is not obvious how one might derive a conservation law or, for that matter, if it even has a conservation law. We developed computational facilities, based on a systematic method, which permits to identify functionals that have conservation laws. The central result used was the celebrated Noether's theorem. This theorem links conservation laws with the invariance properties of the problem (with symmetries), and provides an algorithm for finding conservation laws. Thus the problem is reduced to the one of finding the variational symmetries. We showed how a Computer Algebra System can help to find the symmetries and the conservation laws in the calculus of variations.

For nonsmooth Euler-Lagrange extremals, Noether's conservation laws cease to be valid. We proved that Emmy Noether's theorem of the calculus of variations is still valid in the wider class of Lipschitz functions, as long as one restrict the Euler-Lagrange extremals to those which satisfy the DuBois-Reymond necessary condition. In the smooth case all Euler-Lagrange extremals are DuBois-Reymond extremals, and the result gives a proper extension of the classical Noether's theorem. This is in contrast with the developments of Noether's symmetry theorems to the optimal control setting, which give rise to non-proper extensions when specified for the problems of the calculus of variations.

Particular attention has been done to the important relation between the invariance of an optimal control problem under a family of transformations, and the existence of preserved quantities along the Pontryagin extremals. Several extensions of Noether theorem were provided, in the direction which enlarges the scope of its application. We have obtained a more general version of Noether's theorem for optimal control problems, which incorporates the possibility to consider a family of transformations depending on several parameters and, what is more important, to deal with quasi-invariant and not necessarily invariant optimal control problems. We trust that this latter extension provides new possibilities and we illustrate it with several examples, not covered by the previous

known optimal control versions of Noether's theorem.

We have also obtained a generalization of Noether's invariance principle for optimal control problems with equality and inequality state-input constraints. The result relates the invariance properties of the problems with the existence of conserved quantities along the constrained Pontryagin extremals. A result of this kind was posed as an open question by Vladimir Tikhomirov, in 1986.

We studied Smarandache sequences of numbers and related problems via a Computer Algebra System. Solutions were discovered, and some conjectures presented. In particular, we used the Maple system to check the investigations of S.S. Gupta regarding the Smarandache consecutive and the reversed Smarandache sequences of triangular numbers, extending previous investigations to the mirror and symmetric Smarandache sequences of triangular numbers.

We introduced a new time-dependent definition of spline curves in \mathbb{R}^n , which extends a recent definition of vector-valued splines introduced by Rodrigues and Silva Leite for the time-independent case. Previous results are based on a variational approach, with lengthy arguments, which do not cover the non-autonomous situation. We proved that the previous results are a consequence of the Pontryagin maximum principle, and are easily generalized using the methods of optimal control. Main result obtained asserts that vector-valued splines are related to the Pontryagin extremals of a non-autonomous linear-quadratic optimal control problem.

The mathematical model of a real flexible elastic system with distributed and discrete parameters was studied. The problem is described by a partial differential equation with non-classical boundary conditions. Complexity of the boundary conditions results in the impossibility to find exact analytical solutions and, to address the problem, we investigated the use of the asymptotical method of small parameter together with the numerical method of normal fundamental systems of solutions. These methods allowed us to investigate vibrations, and a technique for determination of complex eigenvalues of the considered boundary value problem was developed. The conditions, at which vibration processes of different character take place, were defined. Dependence of the vibration frequencies on physical parameters of the hybrid system was studied. We showed that introduction of different feedbacks into the system allow one to control the frequency spectrum, in which excitation of vibrations is possible.

2.2 Visiting Appointments of cotg members

May 2004 Delfim Torres has visited SISSA, International School for Advanced Studies, Trieste, Italy (Invited by U. Boscain)

September 2004 Delfim Torres has visited the Department of Control and Automation, Georgian Technical University, Tbilisi (Invited by V. Sesadze)

October 2004 Delfim Torres has visited the University Paris XI, Orsay, France (Invited by E. Trélat)

2.3 Invited Researchers

June 2004 Matthias Kowski, Arizona State University, USA (supported by FLAD fellowship; invited by E. Rocha)

December 2004 Valida Sesadze and Tamar Kekenadze, Georgian Technical University, Tbilisi, Georgia (supported by CEOC; invited by D. Torres)

2.4 Participation in Projects

- Project POCTI/MAT/41683/2001, Advances in Nonlinear Control and Calculus of Variations, FCT – Sapiens’01 (Members of the project: M. Guerra, E. Rocha, D. Torres)
- Research project *Application of Conservation Laws to Space Trajectory*, University of Aveiro and Université Paris-Sud (Delfim F. M. Torres is the responsible for the project together with E. Trélat). The research project is fruit of a collaboration between the University of Aveiro, Portugal, and the University of Orsay, France (Actions Universitaires Intégrées Luso-Françaises, F-30/04).
- Control Training Site (CTS, Marie Curie Fellowships). Delfim F. M. Torres is, from January to June 2005, supervisor of PhD student Ilona Dzenite, from Riga Technical University, Latvia

3 Publications during 2004

3.1 Books

3.2 Articles in International Journals

1. Delfim F. M. Torres, Carathéodory-Equivalence, Noether Theorems, and Tonelli Full-Regularity in the Calculus of Variations and Optimal Control. *Journal of Mathematical Sciences*, Vol. 120, No. 1, 2004, pp. 1032-1050.
2. Delfim F. M. Torres, Quasi-Invariant Optimal Control Problems. *PortugalæMathematica*, Vol. 61, Fasc. 1, 2004, pp. 97-114. [MR 2040245] [Zbl 1042.49015]
3. Delfim F. M. Torres, The Role of Symmetry in the Regularity Properties of Optimal Controls, *Proceedings of Institute of Mathematics of National Academy of Sciences of Ukraine*, Vol. 50, Part 3, pp. 1488-1495, 2004. [MR 2077966]

4. Delfim F. M. Torres, Números Felizes e Sucessões de Smarandache: Digressões com o Maple, *Smarandache Notions Journal*, Vol. 14, 2004, pp. 119–123.
5. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, *WSEAS Transactions on Mathematics*, Issue 3, Vol. 3, July 2004 (ISSN 1109-2769), pp. 620–624. [MR 2089373]
6. Delfim F. M. Torres, Proper Extensions of Noether’s Symmetry Theorem for Nonsmooth Extremals of the Calculus of Variations, *Communications on Pure and Applied Analysis*, Vol. 3, No. 3, 2004, pp. 491–500.
7. Eugénio Rocha, A.Sarychev, A.Pereira, R.Rodrigues. Control-theoretic methods for design of algorithms of digital arithmetic. *Journal of Mathematical Sciences*, vol.120 (I), 2004.
8. Manuel Guerra, Distribution-like Hamiltonian flows and generalized optimal controls. *J. Math. Sci.*, Vol. 120 No 1, 2004, pp 895-918.
9. V. Tydnjuk, V. Kravchenko, Olena Mul, M. Shut, Information Interaction of Thermochemical Potentials in Living Organisms and General Theory of Relativity. *International Scientific Journal of Computing*, Vol. 3, issue 2, 2004, pp.91-98.
10. Paulo D. F. Gouveia and Delfim F. M. Torres, Smarandache Sequences: Explorations and Discoveries with a Computer Algebra System, *Smarandache Notions Journal*, Vol. 14, 2004, pp. 5-22.

3.3 Chapters in Books

3.4 Articles in Nacional Journals

11. Delfim F. M. Torres, Números Felizes e Sucessões Associadas: Digressões com o Maple, *Educação e Matemática* no. 77, Revista da Associação de Professores de Matemática, Março/Abril de 2004, pp. 35–38.

3.5 Proceedings with Referee

12. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, *Proceedings of the 6th WSEAS International Conference on Applied Mathematics (Session on Optimization and Applications)*, Corfu, Greece, August 17-19, 2004 (ISBN: 968-8457-01-7). Paper no. 488-164.
13. Manuel Guerra, Hamiltonian flows for impulsive control systems. Sachkov, Yu. L. (ed): *Generalized solutions in control problems* Proceedings of the IFAC Workshop GSCP-04 and satellite events, Pereslavl-Zalessky, Russia, September 21-29, 2004.

3.6 Research Reports (Preprints)

14. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, *Cadernos de Matemática CM04/I-12*, Dep. Matemática, Univ. Aveiro, 2004. [[arXiv:math.OC/0407409](#)]
15. Delfim F. M. Torres, Rational Neighbor, *Cadernos de Matemática CM04/D-01*, Dep. Matemática, Univ. Aveiro, January 2004. MIUP'2003, 3rd Portuguese ACM Programming Contest.
16. Delfim F. M. Torres, O Jogo do 24 – digressões com o Maple, *Cadernos de Matemática CM04/D-03*, Dep. Matemática, Univ. Aveiro, April 2004.
17. Delfim F. M. Torres and Viorica Teca, Consecutive, Reversed, Mirror, and Symmetric Smarandache Sequences of Triangular Numbers, *Cadernos de Matemática CM04/D-05*, Dep. Matemática, Univ. Aveiro, June 2004. [[arXiv:math.HO/0407112](#)]
18. Eugénio A.M. Rocha, Noncommutative Power Sums Symmetric Functions, *Cadernos de Matemática CM04/I-31*, Dep. Matemática, Univ. Aveiro, 2004.
19. F. Silva Leite, M. Camarinha, Manuel Guerra, Eugénio Rocha, Delfim Torres (eds.), First Control Training Site Workshop – Book of Abstracts, *Cadernos de Matemática CM04/D-07*, Dep. Matemática, Univ. Aveiro, July 2004.
20. Olena V. Mul and Delfim F. M. Torres, Analysis of Vibrations in Large Flexible Hybrid Systems, *Cadernos de Matemática CM04/I-30*, Dep. Matemática, Univ. Aveiro, December 2004.
21. Paulo D. F. Gouveia e Delfim F. M. Torres, Computação Algébrica no Cálculo das Variações: Determinação de Simetrias e Leis de Conservação, *Cadernos de Matemática CM04/I-23*, Dep. Matemática, Univ. Aveiro, September 2004. [[arXiv:math.OC/0411211](#)]
22. Rui C. Rodrigues and Delfim F. M. Torres, Generalized Splines in \mathbb{R}^n and Optimal Control, *Cadernos de Matemática CM04/I-04*, Dep. Matemática, Univ. Aveiro, February 2004. [[arXiv:math.OC/0405200](#)]

3.7 PhD and MSc Thesis

23. Eugénio M. Rocha, An Algebraic Approach to Nonlinear Control, PhD Thesis, Univ. of Aveiro, 2004.

3.8 Accepted Papers

24. Delfim F. M. Torres, A Noether Theorem on Unimprovable Conservation Laws for Vector-Valued Optimization Problems in Control Theory, Accepted (paper ID: 131) in the Conference Proceedings of ICMSAO'05 – First International Conference on Modeling, Simulation and Applied Optimization, February 1-3 2005, American University of Sharjah, United Arab Emirates, 2005. [[arXiv:math.OA/0411173](#)]
25. Delfim F. M. Torres and Viorica Teca, Consecutive, Reversed, Mirror, and Symmetric Smarandache Sequences of Triangular Numbers. Accepted to Smarandache Notions Journal.
26. Manuel Guerra, Discontinuous Hamiltonian flows for nonlinear control systems, Rend. Sem. Mat. Univ. Pol. Torino, accepted March 2004.
27. Rui C. Rodrigues and Delfim F. M. Torres, Generalized splines in \mathbb{R}^n and optimal control. Accepted to Rend. Sem. Mat. Univ. Pol. Torino.

4 Talks during 2004

4.1 Talks at International Conferences

1. Cristiana J. Silva and Delfim F. M. Torres, On the Classical Newton's Problem of Minimal Resistance, Third Junior European Meeting on "Control, Optimization and Computation", University of Aveiro, Portugal, September 6–8, 2004.
2. Delfim F. M. Torres, Symmetry in the Calculus of Variations and Optimal Control, Young European Researchers in Mechanics Meeting 2004 (YERM2004), Instituto Superior Técnico, Lisbon, Portugal, January 2004.
3. Delfim F. M. Torres, On the Noether Invariance Principle for Constrained Optimal Control Problems, 6th WSEAS International Conference on Applied Mathematics (Session on: Optimization and Applications), Corfu, Greece, August 17-19, 2004.
4. Delfim F. M. Torres, Lipschitzian Regularity of the Minimizing Trajectories in the Calculus of Variations and Optimal Control: a Survey, Third Junior European Meeting on "Control, Optimization and Computation", University of Aveiro, Portugal, September 6–8, 2004.
5. Eugénio Rocha, A reduction method for SR-geodesics on nilpotent Lie groups, Fifth International Conference on Optimization, Lisbon, Jul/2004.
6. Eugénio Rocha, A transition relation between some noncommutative symmetric functions, 11th Conference of the International Linear Algebra Society, Coimbra, Jul/2004.

7. Eugénio Rocha, Formal Control Theory and Applications, Third Junior European Meeting on “Control, Optimization and Computation”, University of Aveiro, Portugal, September 6–8, 2004.
8. Manuel Guerra, Discontinuous Hamiltonian flows and generalized extremals for nonlinear control systems. Workshop “Geometry of Control Systems and Distributions”. Banach Center, Polish Academy of Science, Warsaw, 17 de Maio de 2004.
9. Manuel Guerra, Generalized synthesis for singular nonlinear control systems. 3rd Junior European Meeting, University of Aveiro, 6-8 September 2004, Portugal
10. Manuel Guerra, Hamiltonian flows for impulsive control systems. IFAC Workshop GSCP-04, Pereslavl-Zalessky, Russia, September 21-29, 2004.
11. Olena Mul, On Control of Vibrations in the Complex Dynamical Systems of Machine Units. The Fifth International Conference on Optimization “Optimization 2004”, Lisbon, Portugal, July 2004.
12. Olena Mul, Control of Vibrations in Some Hybrid Systems of Controlled Machine Units, Third Junior European Meeting on Control, Optimization and Computation, Aveiro, Portugal, September 2004.
13. Olena Mul, Differential Equations in Some Applications of Systems Analysis. The conference “CR Geometry and Partial Differential Equations”, Levico Terme, Trento, Italy, September 2004.
14. Paulo D. F. Gouveia and Delfim F. M. Torres, A Computer Algebra Package for Determining Symmetries and Conservation Laws in the Calculus of Variations, Invited Session: Optimal Control I, Optimization 2004, Lisbon, Portugal, 25 - 28 July 2004.
15. Paulo D. F. Gouveia e Delfim F. M. Torres, Computação Algébrica no Cálculo das Variações: Determinação de Simetrias e Leis de Conservação, Congresso Nacional de Matemática Aplicada e Computacional, XXVII CNMAC, (Brazilian Congress of Applied Mathematics and Computation), Porto Alegre, Brasil, 13-16, September 2004.
16. Rui C. Rodrigues and Delfim F. M. Torres, Linear-quadratic optimal control problems and spline functions in Euclidean spaces, First CTS Workshop, Coimbra, Portugal, July 2004.

4.2 Talks at National Conferences

4.3 Seminars

17. Delfim F. M. Torres, Da Adrenalina em Escorregas e Condução de Helicópteros à Exploração Marciana, Conferências de Matemática, Ciclo de conferências

destinadas a toda a comunidade universitária com interesse nos avanços da Matemática, Dep. Matemática, Univ. Aveiro, 28 de Janeiro de 2004. (Invited Talk, in representation of the research group *cotg* – Control Theory Group).

18. Delfim F. M. Torres, O Jogo do 24 – digressões com o Maple, Conferência destinada a Estagiários e Orientadores de Estágio, Dep. Matemática, Univ. Aveiro, 29/Março/2004
19. Eugénio Rocha, Formal chronological calculus and noncommutative symmetric functions, CEOC, Aveiro, Nov/2004.
20. Eugénio Rocha, Integrability of sub-Riemannian geodesics on a nilpotent Lie group, ISR, Univ. of Coimbra, Mar/2004.
21. Manuel Guerra, Geometric control theory: some singular cases, CEMAPRE/ISEG, November 16, 2004

5 Organization of Conferences during 2004

1. First Control Training Site Workshop (1stCTSW), Department of Mathematics, University of Coimbra, Portugal, 1–3 July 2004, Portugal (Members of the Organizing Committee: M. Guerra, E. Rocha, and D. Torres)
2. 3rd Junior European Meeting on Control, Optimization, and Computation, University of Aveiro, 6–8 September 2004, Portugal (Members of the Organizing Committee: M. Guerra, O. Mul, E. Rocha, D. Torres; Chairman: D. Torres)

6 Organization of Special/Invited Sessions during 2004

1. Optimal Control I, Optimization 2004 (organizers: D. Torres, E. Rocha, and M.d.R. de Pinho). Participants: M.M.A. Ferreira, F.A.C.C. Fontes, P.D.F. Gouveia, O. Mul, M.d.R. de Pinho, R.C. Rodrigues, F. Silva Leite, and D.F.M. Torres.
2. Optimal Control II, Optimization 2004 (organizers: E. Rocha, and D. Torres). Participants: R.M. Bianchini, M.I. Caiado, G. Dirr, U. Helmke, K. Hüper, M. Kleinsteuber, Y. Liu, E.M. Rocha, and A.V. Sarychev
3. Mathematical Control Theory I, Controlo 2004 (organizers: M. d. R. de Pinho, M. M. A. Ferreira, F. A. C. C. Fontes, and D. Torres). Participants: F. Clarke, P. Cannarsa, G. Fragnelli, J. Vancostenoble, A. Rapaport, and P. Cartigny

4. Mathematical Control Theory II, Controlo 2004 (organizers: M. d. R. de Pinho, M. M. A. Ferreira, F. A. C. C. Fontes, and D. Torres). Participants: G. Colombo, P. R. Wolenski, J. Rosenblueth, H. Maurer, J.-H. R. Kim, A. Yu. Plakhov, and D. F. M. Torres
5. Degeneracy Phenomena in Optimal Control and Calculus of Variations, Controlo 2004 (organizers: A. C. de Matos, M. d. R. de Pinho, and D. Torres). Participants: M. M. A. Ferreira, F. A. C. C. Fontes, S. O. Lopes, M. d. R. de Pinho, A. Arutyunov, V. Dykhta, and F. Pereira.

7 Scientific Indicators of 2004

		Expected
Books	0	0
Papers in International Journals	10	5
Chapters in Books	0	
Papers in National Journals	1	1
Total Number of Publications	11	
Proceedings with Referee	2	
Accepted Papers	4	
Research Reports (Preprints)	9	0
Talks in International Meetings	16	6
Talks in National Meetings	0	0
Total Number of Communications	16	
Talks at Seminars	5	
Organization of International Conferences	2	2
Number of PhD theses under Supervision	1	
Number of Master theses under Supervision	3	
Total number of Advanced Training under Supervision	4	
Number of PhD theses completed	1	1
Number of Master theses completed	0	0
Total number of Advanced Training completed	1	