CEOC

Centro de Estudos em Optimização e Controlo
(Centre for Research in Optimization and Control)

Problems of Minimal Resistance and Problems of Mass Transfer

Research Plan 2003-2005

Universidade de Aveiro
Departamento de Matemática

Research Plan 2003-2005

1. Problems of minimal resistance and problems of mass transfer

1.1 Research team

- **Coordinator**: Alexander Plakhov
- **Researchers**:
  1. Alexander Plakhov (PhD, 20%)
  2. Delfim Torres (PhD, 10%)
  3. Leslie Bajuelos (PhD, 5%)
  4. Tatiana Tchemisova (PhD, 10%)
  5. Evgeny Lakshatanov (PhD, 100% em 2006)
- **Scientific cooperation**:
  1. Anatoly Stepin, Moscow State University
  2. Vladimir Levin, Central Institute of Economics and Mathematics, Moscow
  3. Thomas Lachand-Robert, Savoie University, France
  4. Diogo Gomes, Instituto Superior Técnico
  5. Giuseppe Buttazzo, University of Pisa, Italy

1.3 Project's summary

To study the problem of minimal resistance for the bodies moving in a rarefied medium, as well as to study the exactly solvable problems of mass transfer and to investigate the relation between finite- and infinite-dimensional transport problems.

1.4 Project's description objectives

Study of the classical (Newtonian) problem of the body of least resistance in various classes of bodies: non-convex axisymmetric and non-axisymmetric bodies (two- and three-dimensional cases, with and without restrictions imposed on the maximal number of collisions of particles with the body). Minimization of specific resistance of infinite surfaces. Minimization of resistance to bodies moving in a medium of positive temperature, herein including bodies which perform both translational and rotational motions, and of rotating bodies with rough surface. Relationship between the problem of minimal resistance of non-convex rotating bodies and the one-dimensional problem of mass transfer. Exactly solvable problems of mass transfer; relationship between linear transport and mass transfer problems. The problem of finding the pressure generated by a given non-parallel flux of particles and the inverse problem of determining density of the flux generating a given pressure distribution.

1.5 Relationship with the state of the art

Since the early 1990th, the interest to Newton's problem of the body of minimal resistance revived. By means of calculus of variations, interesting and unexpected results have been obtained for the classes of non-axisymmetric and/or non-convex bodies (recall that Newton stated his problem for convex axisymmetric bodies). See papers of Buttazzo, Kawohl, Lachand-Robert, and others. Throughout this study, it was assumed that every particle of the medium collides with the body at most once. On the contrary, the most part of our research is devoted to the case of multiple collisions of particles with the body. This problem seems to be more realistic; it is found to be closely related to the billiard theory and the mass transfer problem. Newton assumed the collisions to be absolutely elastic and the particles to be immovable. Horstmann et al (1999) considered the case of non-elastic collisions, which is physically more relevant. We expect to advance in this direction, considering resistance in a medium with temperature motion of particles. The theory of mass transfer is rapidly growing last years; yet, the number of exactly solvable problems, even in one dimension, is relatively small. Also, the relation between finite-dimensional transport problems and the mass transfer problems is not completely understood. Our aim is to discover new exactly solvable problems, as well as to represent the mass transfer problem as the limiting case of finite-dimensional transport problem. The problems of resistance of rotating and/or rough bodies were not studied. Yet, they may be of interest when constructing artificial satellites.

1.6 Expected indicators

Number of Publications	**2003**	**2004**	**2005**	**Total**
Books	0	0	0	0
Papers in international journals	2	5	4	11
Papers in national journals	0	0	0	0
Number of Communications	**2003**	**2004**	**2005**	**Total**
in International Meetings	2	3	3	8
in National Meetings	0	2	4	6
Reports	5	3	3	11
Organization of seminar and conferences	2	3	3	8
Advanced training	**2003**	**2004**	**2005**	**Total**
number of PhD theses	0	0	0	0
number of Master theses	0	0	0	0

1.7 Tasks

1.7.1 Minimal resistance and problems of mass transfer

- **Task duration - months**: 36
- **persons*month**: 16,2
  - **Research team**: Alexander Plakhov (20%), Delfim Torres (10%) and Leslie Bajuelos (5%) Tatiana Tchemisova (10%).
- **Expected results** (max. 100 words): Solution of Newton's problem in the classes of non-convex axisymmetric and non-axisymmetric bodies, in two and three dimensions. Zero-resistance solution. Solution of the problem in arbitrary dimension for bodies containing a half-space. The body of minimal resistance in a medium of positive temperature; limiting cases of zero and of infinite temperature. Estimation of resistance of convex and non-convex rotating bodies, in two and three dimensions, and of rotating bodies with rough surface. Solution of the direct and inverse problems of non-parallel flux of particles. Exact solutions of a one-dimensional mass transport problem with cost function having an interval of convexity and an interval of concavity. Find exact results of some geometrical two-dimensional mass transfer problems.
- **Task description** (max. 200 words): The results on Newton's problem of minimal resistance will be obtained using methods of billiard theory, of mass transfer, and of optimal control theory. It is planned to obtain exact solutions on mass transfer in two dimensions, using methods of finite-dimensional transport problem.