

CEOC

Centro de Estudos em Optimização e Controlo
(Centre for Research in Optimization and Control)

**Problems of Minimal Resistance and
Problems of Mass Transfer**

Scientific Reports 2003-2005

Universidade de Aveiro
Departamento de Matemática

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Scientific report 2003

1. Problems of minimal resistance and problems of mass transfer

1.1 Activities during 2003

Newton's problem of minimal resistance is formulated as follows. A body is moving through a rarefied medium. It is required to find shape of the body minimizing resistance of the medium to the motion of body. Newton solved this problem in the class of convex axisymmetric bodies. In the early 1990's the interest in the problem revived; interesting minimization results have been obtained in the classes of non-convex and/or non-axisymmetric bodies (see works of Buttazzo, Kawohl, Lachand-Robert and others). All these results were obtained under the assumption that every particle collides with the body at most once. In 2003, we considered the problem in the case where multiple collisions are allowed. It is proved that resistance of an arbitrary connected body can be made less than any positive number, by arbitrarily small deformation of its boundary. Furthermore, the problem of minimal resistance for non-convex non-axisymmetric bodies with fixed height and fixed horizontal circular cross section was considered. It is proved that if double and triple collisions are allowed, the infimum of resistance is zero. A two-dimensional version of the problem was considered.

1.2 Output indicators

Number of Publications	2003
Books	0
Papers in international journals	2
Papers in national journals	0
Number of Communications	
in International Meetings	2
in National Meetings	0
Reports	5
Organization of seminar and conferences	2
Advanced training	
number of PhD theses	0
number of Master theses	0

1.3 List of publications

- **Articles in International Journals (including book chapters)**

1. A.Yu. Plakhov, "Newton's problem of the body of minimal aerodynamic resistance", Doklady of the Russian Academy of Sciences 390, 3 (2003): 1-4.
2. A. Yu. Plakhov, Newton's problem of the body of minimal resistance with a bounded number of collisions, Russ. Math. Surv. 58, 1 (2003): 191-192.

1.4 List of talks

- **Talks at International Conferences**

1. A.Yu. Plakhov, "Newton's problem of minimal averaged resistance in the class of bodies of fixed volume", Int. Conf. "Kolmogorov and contemporary mathematics", June 16-21, 2003, Moscow, Russia.
2. Delfim F. M. Torres and Alexander Plakhov, "Optimal control of Newton-type problems of minimal resistance", Second Junior European Meeting, Control Theory and Stabilization, December 2003, Torino, Italy,

1.5 List of reports (including proceedings)

- 1. A.Yu. Plakhov, "Newton's problem of the body of minimal aerodynamic resistance", Cadernos de Matemática, Universidade de Aveiro, CM03/I-05, (2003): 6 p.
 2. A.Yu. Plakhov, "Newton's problem of the body of least resistance: the case of few impacts", Cadernos de Matemática", Universidade de Aveiro, CM03/I-07, (2003): 7 p.

3. A.Yu. Plakhov, "Newton's problem of minimal resistance for bodies containing a half-space", *Cadernos de Matemática, Universidade de Aveiro, CM03/I-21*, Universidade de Aveiro, (2003): 4 p.
4. A.Yu. Plakhov, "Newton's problem of the body of minimal averaged resistance", *Cadernos de Matemática, Universidade de Aveiro, CM03/I-34*, (2003): 24 p.
5. A.Yu. Plakhov, "Exact solutions of the one-dimensional Monge-Kantorovich problem", *Cadernos de Matemática, Universidade de Aveiro, CM03/I-36*, (2003): 18 p.

1.6 List of organized seminars and conferences

- **Organized seminars**

1. Alexander Plakhov, "O problema de Newton de resistência mínima e o problema de Monge de transporte de massa: dois problemas antigos revisitados", *Seminários do CEOC*, October 3, 2003, Universidade de Aveiro.
2. Alexander Plakhov, "O problema de Newton do corpo cuja resistência média é mínima", *Seminários do CEOC*, May 9, 2003, Universidade de Aveiro.

Scientific report 2004

2. Problems of minimal resistance and problems of mass transfer

2.1 Activities during 2004

- The study of Newton's problem of the body of least resistance in the case of multiple collisions was continued. The problem of minimization of resistance (per unit surface area) was solved in arbitrary dimension. The problem of resistance of rough bodies (with and without rotation) was stated. Some estimates, based on the Monge-Kantorovich mass transportation problem, were obtained in two- and three-dimensional cases. As a "by-product" of this activity, the Monge-Kantorovich problem in one dimension was studied, and some new exact solutions were obtained. These solutions are related to the case where cost function has intervals of both convexity and concavity. During 2004 the following researcher, with joint work with members of this project, visited our Mathematics Department.

1. Thomas Lachand-Robert, University of Savoie.
2. Anatole Stepin, Moscow State University.
3. Evgeny Lakshtanov, Moscow State University.
4. Vladimir Levin, CEMI (Central Institute of Economics and Mathematics), Moscow.

2.2 Output indicators

Number of Publications	2004
Books	0
Papers in international journals	3
Papers in national journals	0
Number of Communications	
in International Meetings	2
in National Meetings	0
Reports	2
Organization of seminar and conferences	2
Advanced training	
number of PhD theses	0
number of Master theses	0

2.3 List of publications

- **Articles in International Journals (including book chapters)**
 1. A.Yu. Plakhov, Newton's problem of minimal resistance for bodies containing a half-space. *J. Dynam. Control Syst.*, **10**, N°2 2004: 247-251.
 2. A.Yu. Plakhov, Newton's problem of the body of minimum mean resistance. *Sbornik: Mathematics*, **195**, N°7 (2004): 1017-1037.
 3. A.Yu. Plakhov, Precise solutions of the one-dimensional Monge-Kantorovich problem. *Sbornik: Mathematics*, **195**, N°9 (2004): 1291-1308.

2.4 List of talks

- **Talks at International Conferences**
 1. Alexander Yu. Plakhov, Delfim F. M. Torres, Two-dimensional problems of minimal resistance in a medium of positive temperature, Controlo-2004, Universidade de Faro, Faro, Portugal, June 2004.
 2. A.Yu. Plakhov, The problem of minimal and maximal resistance of rotating bodies, 4th Conference on Diff. Eqs. and Dynam. Systems, Suzdal (Russia), July 2004.
- **Seminars**
 1. Alexander Yu. Plakhov,, *Newton's problem of minimal resistance and Accelerated algorithms of stochastic approximation (review)*, 30/01/2004, Seminário do CEOC, Universidade de Aveiro.
 2. Alexander Yu. Plakhov, *Newton's aerodynamical problem in media of positive temperature*, 14/10/2004, Seminário do CEOC, Universidade de Aveiro.

3. Alexander Yu. Plakhov, *The problem of minimal resistance of a rotating body and problems of mass transport*, 21/05/2004, Instituto Superior Técnico.
4. Alexander Yu. Plakhov, *O problema de Newton de resistência mínima: caso de colisão es múltiplas*, 20/02/2004, Instituto Superior Técnico.
5. Alexander Yu. Plakhov, *Os problemas de resistência mínima*, 14/05/2004, Universidade do Porto.
6. Alexander Yu. Plakhov, *Problem of shape optimization for bodies moving in rarefied media*, 4/11/2004, Seminar on Dynamical Systems, Moscow State University.
7. *Problems of the body of least averaged resistance*, 19/01/2004, University of Savoie.

2.5 List of reports (including proceedings)

- 1. Delfim F. M. Torres and Alexander Plakhov, Optimal control of Newton-type problems of minimal resistance. Cadernos de Matemática CM04/I-01, Dep. Matemática, Univ. Aveiro, January 2004.
- 2. Alexander Yu. Plakhov and Delfim F. M. Torres, Two-dimensional problems of minimal resistance in a medium of positive temperature. Cadernos de Matemática CM04/I-08, Dep. Matemática, Univ. Aveiro, 2004 (published in Proceedings of 6th Portuguese Conference on Automatic Control - controlo2004, p. 488-493, 2004).

2.6 List of organized seminars and conferences

- **Organized seminars**
 1. Diogo Aguiar Gomes (Universidade Técnica de Lisboa), "Métodos de Equações Diferenciais Parciais em Mecânica Clássica", June 25, 2004, Universidade de Aveiro.
 2. Alexander Plakhov e Delfim Torres, "Newton's aerodynamical problem in media of positive temperature", October 15, 2004, Universidade de Aveiro.

Scientific report 2005

3. Problems of minimal resistance and problems of mass transfer

3.1 Activities during 2005

The work continued in several directions.

First, a Newton-type problem of minimal resistance was studied in media with *thermal motion of particles*, in three-dimensional and two-dimensional cases. A classification of solutions (optimal shapes) was made; in an important particular case of *homogeneous monatomic ideal gas* there were numerically constructed diagrams in the parameter space, indicating the correspondence between parameters of the system and optimal shapes. An *universal* optimal shape was found in the infinite-temperature limit.

On the other hand, the study of rotating bodies in two dimensions was continued. This time, we considered bodies performing *incomplete rotations* with respect to the direction of motion. The problem of minimal averaged resistance was reduced to a special class of one-dimensional Monge-Kantorovich problems. These problems were first solved numerically, and then, analytically.

Further, the study of a quantum analogue of Newton's problem was started. The problem was stated in terms of transport cross section, and some estimates for low frequencies were obtained.

Finally, there was made a research in the theory of cellular automata (CA). Last time, CA are used as a discrete model of processes usually described by PDE (justification of such an approach can be used, e.g., in: Theory of Applications of Cellular Automata, edited by S. Wolfram (World Scientific, Singapore, 1986)). A possibility of application of CA for studying the processes of wave scattering on obstacles was investigated; in particular, there was made a research on entropy of CA.

During 2005, the following mathematicians, whose research interest are focused on problems of minimal resistance or on problems of mass transfer, visited our Mathematics Department.

1. Robert McCann, University of Toronto (Canada).
2. Thomas Lachand-Robert, University of Savoie (France).
3. Myriam Comte, University of Paris VI (France).
4. Evgeny Lakshtanov, Moscow State University (Russia).

3.2 Output indicators

Number of Publications	2005
Books	0
Papers in international journals	3
Papers in national journals	0
Number of Communications	
in International Meetings	5
in National Meetings	1
Reports	3
Organization of seminar and conferences	5
Advanced training	
number of PhD theses	0
number of Master theses	1

3.3 List of publications

- **Articles in International Journals (including book chapters)**

1. A.Yu. Plakhov, Bodies of minimal aerodynamic resistance in dilute media with thermal motion of the particles, Dokl. Akad. Nauk v.403 1, (2005): 15-19.
2. A. Yu. Plakhov and Delfim F. M. Torres, Newton's aerodynamic problem in media of chaotically moving particles, Sbornik: Mathematics v.196 5-6, (2005): 885-933.
3. A. Yu. Plakhov, On the billiards in unbounded regions, inverting the direction of particles' motion, approved for publication in Russ. Math. Surv., August 2005.

3.4 List of talks

- **Talks at International Conferences**

1. A. Yu. Plakhov and T. V. Tchemisova, "On application of discrete optimization methods to solving a mass transfer problem", Int. Conf. "Control Problems and Applications (technology, industry, economics)", Apr. 15-19, 2005, Minsk, Belarus.

2. A. Yu. Plakhov, "Billiards and problems of minimal aerodynamic resistance", XXIV Workshop on Geometric Methods in Physics, June 26 - July 2, 2005, Białowieża, Poland.
3. A. Yu. Plakhov and T. V. Tchemisova, "On discrete simulation of mass transportation", IFORS 2005, July 11 - 15, 2005, Hawaii.
4. A. Yu. Plakhov, "Newton's problem of minimal resistance and its generalizations", The 4th Int. Conf. on Diff. and Func. Diff Eqs., Aug. 14-21, 2005, Moscow, Russia.
5. A. Yu. Plakhov, "Optimal mass transportation and problems of minimal resistance", MSRI Workshop "Optimal Mass Transport and its Applications", Nov. 14-18, 2005, Berkeley, USA.

- **Talks at National Conferences**

1. A. Yu. Plakhov, *Problemas de resistência mínima*, CEOC/CIMA-UE joint meeting on Optimization and Optimal Control, Évora, Portugal, April 22-23, 2005.

- **Seminars**

1. A. Yu. Plakhov, *Aerodynamic problems in rarefied media*, IST, Lisbon, March 3, 2005.
2. A. Yu. Plakhov, *Problems of least resistance and billiards*, Seminar conducted by Acad. Ya.G. Sinai, Moscow, July 27, 2005.
3. A. Yu. Plakhov, *Shapes of minimal and maximal aerodynamic resistance*, CAM seminar, University of St. Thomas, Minneapolis, USA, November 21, 2005.

3.5 List of reports (including proceedings)

- 1. A.Yu. Plakhov, "Billiards inverting the direction of particles' motion", *Cadernos de Matemática*, CM05/I-29 Universidade de Aveiro (2005): 9 p.
 2. A.Yu. Plakhov, "The problem of minimal and maximal averaged resistance of moving bodies", *Cadernos de Matemática*, CM05/I-08, Universidade de Aveiro (2005): 10 p.
 3. A. Yu. Plakhov and Delfim F. M. Torres, "Newton's aerodynamic problem in a medium consisting of moving particles", *Cadernos de Matemática*, CM04/I-22, Universidade de Aveiro (2005): 47 p.

3.6 List of organized seminars and conferences

- **Organized seminars**

1. T. Lachand-Robert (University of Savoy, France), "Optimization with convexity constraints: numerical methods", Seminários do CEOC, Universidade de Aveiro, January 11, 2005.
2. Robert McCann (University of Toronto, Canada), "Optimal Transportation with costs changing concavity", Seminários do CEOC, Universidade de Aveiro, January 12, 2005.
3. Evgeny L. Lakshtanov (Institute for Information Transmission Problems, Moscow, Russia), "Quantum analogue of Newton's problem of the body of least resistance", Seminários do CEOC, Universidade de Aveiro, March 18, 2005.
4. A. Yu. Plakhov, P. Gouveia, *Alguns problemas de optimização em bilhares: resultados analíticos e numéricos*, Seminário do CEOC, Universidade de Aveiro, May 20, 2005.
5. A. Yu. Plakhov, *Aerodinâmica do movimento em meios rarefeitos e transporte de massa*, Seminário do CEOC, Universidade de Aveiro, December 2, 2005.

3.7 List of PhD and MSc dissertations

- MSc dissertations

1. Susana Raquel da Silva Leal Pereira, "Integrabilidade e dispersão de sistemas Hamiltonianos", Universidade de Aveiro, October 2005 (supervisor: Alexander Plakhov).