CEOC

Centro de Estudos em Optimização e Controlo
(Centre for Research in Optimization and Control)

Optimization, Graph Theory and Combinatorics

Research Plan 2003-2005

Universidade de Aveiro
Departamento de Matemática

Research Plan 2003-2005

1. Optimization, graph theory and combinatorics

1.1 Research team

- **Coordinator**: Domingos Moreira Cardoso
- **Researchers**:

1. Agostinho Agra (PhD, 30%)
2. Alexander Plakhov (PhD, 20%)
3. António Batel Anjo (PhD, 30%)
4. Carlos J. Luz (PhD, 30%)
5. Cristina Requejo (PhD, 30%)
6. Domingos Moreira Cardoso (PhD, 40%)
7. Eugénio Rocha (PhD, 10%)
8. Fátima Pacheco (MSc, PhD student, 100% em 2006)
9. João Pedro Cruz (MSc, PhD student, 100%)
10. Natália Martins (PhD student, 50%)
11. Nuno Baeta (PhD student, 30% em 2006)
12. Paula Carvalho (PhD, 40%)
13. Paula Oliveira (PhD, 30%)
14. Paula Rama (MSc, PhD student, 35%)
15. Ricardo Almeida (PhD student, 50%)
16. Rosa Amélia Martins (PhD, 30%)
17. Sofia Pinheiro (MSc, PhD student, 30% em 2006)
18. Tatiana Tchemisova (PhD, 15%)
19. Vitor Neves (PhD, 30%)

- **Scientific cooperation**:

  1. António Guedes de Oliveira, Universidade do Porto
  2. Charles Delorme, University of Paris-Sud, Orsay, France
  3. Dieter Rautenbach, Tcchnische Universität Ilmenau, Ilmenau, Deutschland
  4. Dragos Cvetkovic, University of Belgrade, Belgrade, Serbia and Monte Negro
  5. Imme van den Berg, Universidade de Évora
  6. Jerzy Szymanski, Adam Mickiewicz University, Poznan, Poland
  7. João F. Queiró, Universidade de Coimbra
  8. Jorge F. Sousa, Universidade do Porto
  9. Luís A. Vieira, Universidade do Porto
  10. Luís Gouveia, Universidade de Lisboa
  11. Maria João Borges, Instituto Superior Técnico
  12. Miguel Constantino, Universidade de Lisboa
  13. Mohammad Rostami, Universidade da Beira Interior
  14. Rommel Barbosa, Universidade Federal de Goiás, Goiânia, Brazil

1.3 Project's summary

Within this project we deal with analytic, algebraic and combinatorial approaches to several problems in graph theory and combinatorics. Special attention is given to the following topics: spectra of graphs, mainly eigenvectors and eigenvalues of graphs with regularity constraints and their relationship to some subsets of vertices or edges with particular properties; combinatorial properties of the least eigenvalue with applications to the recognition of graphs for which the stability number may be determined by solving a convex quadratic programming problem; improvements on lower and upper bounds on the stability number of graphs, obtained by convex quadratic programming, and comparisons with the Lovász theta number; extensions and applications of the above results to combinatorial optimization. We also intend to study Jordan algebras and their applications to conic optimization (namely, self-concordant barriers and their parameters) and the abnormal extrema phenomenon in nonlinear programming. We expect to obtain results on combinatorics and related topics, namely in multiattribute ranking problems and also on strongly regular graphs, particularly about the existence of the fourth graph of Moore. Stochastic optimization and stochastic approximation algorithms will be developed as well as applications of nonstandard analysis to optimization and control. Furthermore, we intend to provide advanced training in optimization and discrete mathematics and to develop intership programs and research collaboration with industry.

1.4 Project's description objectives

The primary goals of this project include carrying out research and providing advanced training in optimization, graph theory and combinatorics. Aiming to capture the interest of Portuguese students and junior researchers to discrete mathematics, a Portuguese book on this topic will be published (as far as we know this will be the first book in this language). Additionally, in co-operation with other groups and universities, a MSc course on discrete mathematics will be prepared. The main research objectives cover *graph spectra*, *continuous optimization and graphs*, *combinatorics and related topics*, *stochastic optimization and stochastic approximation* and *nonstandard optimization techniques*. We search for practical implementation of results of our work and for further development of internship programs in cooperation with industry.

1. **Objectives on graph spectra**. We expect to obtain results about adjacency and Laplacian eigenvectors and eigenvalues of graphs with regularity constraints and about their relations with the existence of subsets of vertices and subsets of edges with regularity properties.

2. **Objectives on continuous optimization and graphs**. Recognition of graphs with convex-QP stability number (that is, for which the stability number may be determined by convex quadratic programming), improvements on the determination of lower and upper bounds on the stability number of graphs obtained by convex quadratic programming and extensions to combinatorial optimization problems. Furthermore we intend to apply Jordan algebras on self-concordant barriers over symmetric cones and to study necessary and sufficient conditions for abnormal extrema without constraint qualification.

3. **Objectives on combinatorics and related topics**. Results about the existence of the fourth graph of Moore. Conditions under which a ranking solution is a linear extension of a quasi-order extension of a preference weighted sum relation defined by a multiattribute ranking problem. Concordance graphs and concordance pairs of linear orders. Application of specialization orders and digital topologies in shape reorganization in digitalized image.

4. **Objectives on stochastic optimization and stochastic approximation**. Algorithms for the stochastic optimal path problem. Algorithms of accelerated convergence, aiming to achieve better performance by speeding up the transient stage in stochastic approximation.

5. **Objectives on nonstandard optimization techniques**. To apply hyperfinite discretization and non-standard hull methods in critical point theory as well as to control in differentiable manifolds.

1.5 Relationship with the state of the art

1. **Graph spectra**. Spectra of graphs is an intensive research area with a few results on graphs with regularity constraints [?, ?]. A graph with a (k, τ)-regular set (a set of vertices S inducing a k-regular graph such that each vertex out of S has τ neighbors in S) is an example of a graph with regularity constraints. Several practical problems can be model by (k, τ)-regular sets. For instance, a graph G is Hamiltonian if and only if its line graph has a $(2, 4)$-regular set inducing a connected graph. Another example, a graph has a perfect matching if and only if its line graph has a $(0, 2)$-regular set.

2. **Continuous optimization and graphs**. The recognition of graphs with convex quadratic stability number [?] still remains an open problem. Furthermore only a few computational experiments about lower and upper bounds on the stability number of graphs determined by convex quadratic programming were done [?, ?]. On the other hand, the relationship of convex quadratic bounds for the stability number with the Lovasz theta number [?] still needs clarification. Several approaches of conic optimization using Jordan algebras [?] can be simplified and some extensions to combinatorial problems can be obtained . Concerning the abnormal extrema [?] there are no necessary and sufficient conditions for the occurrence of this kind of phenomena.

3. **Combinatorics and related topics**. There is a very short family of strongly regular graphs with parameters $(n, k; 0, 1)$, with 3 known graphs. It is not known whether there exists one additional strongly regular graph with parameters $(3250, 57; 0, 1)$ designated fourth graph of Moore [?]. There are several results related with its possible existence. On the other hand, validation techniques in multiattribute ranking problems using weak order extensions of posets are not common on the literature. Furthermore, there are several open problems related with concordance graphs [?] which are the comparability graphs of posets of dimension two with particular combinatorial properties. Specialization orders and digital topologies is a hot topic with several applications in image processing and image recognition [?].

4. **Stochastic optimization and approximation**. The stochastic optimal path problem is being studied since 1969. However, only a few theoretical results are known, even though there is a recognizable applicability to telecommunication networks. The proposed mathematical model maximizes the expected value of an utility function over a directed network, where the distances related with the arcs are real random variables following gaussian distributions.
   A stochastic approximation algorithm with step size adaptation was proposed and its convergence was studied [?]. Now, an algorithm with geometric step size adaptation is proposed and its convergence is proved.

5. **Nonstandard optimization techniques**. Applications of nonstandard analysis to critical point theory have not yet been made; in 1993 and 1994 Tuckey ([?]) and Rubio ([?]) have used hyperfinite techniques in variational calculus to treat Young's generalized curves and turbulence problems and in order to obtain mild generalizations of variational principles. Attempts at a nonstandard analysis of manifolds have been made since the late 1970s until 2000, with mixed success, by Stroyan (study of curvature in [?]), Neves (topology on spaces of smooth functions in [?]), Schlesinger (dynamical systems and twistor theory in [?]) and Hertrich-Jeromin (griding of surfaces in [?]).

1.6 Expected indicators

Number of Publications	**2003**	**2004**	**2005**	**Total**
Books	1	0	1	2
Papers in international journals	4	11	15	30
Papers in national journals	1	0	0	1
Number of Communications	**2003**	**2004**	**2005**	**Total**
in International Meetings	10	15	17	42
in National Meetings	2	0	0	2
Reports	9	4	7	20
Organization of seminar and conferences	10	13	15	38
Advanced training	**2003**	**2004**	**2005**	**Total**
number of PhD theses	0	4	4	8
number of Master theses	7	2	0	9

1.7 Tasks

1.7.1 Graph spectra

- **Task duration - months**: 36
- **persons*month**: 28,8
  - **Research team**: Domingos M. Cardoso (15%), Paula Rama (35%) and Rosa Amélia Martins (30%).
- **Expected results** (max. 100 words): Characterization of (k, τ)-regular sets using adjacency and Laplacian eigenvectors and eigenvalues. Spectral implications of the existence of (k, τ) regular sets in graphs. Spectral results on graphs with equitable and/or almost equitable partitions. Characterization of graphs whose minimum eigenvalue does not decrease when an edge or a node is deleted.
- **Task description** (max. 200 words): The task is based on the study of adjacency and Laplacian eigenvectors and eigenvalues of graphs and on their relations with particular combinatorial structeres. The study also covers

graph products and its eigenvalue and eigenvector implications. Particular attention will be given to the minimum eigenvalue of several families of graphs which we call adverse (these are graphs where the minimum eigenvalue does not change when any vertex or the neighborhood of any vertex is deleted).

1.7.2 Continuous optimization and graphs

- **Task duration - months**: 36
- **persons*month**: 41,4
  - **Research team**: Agostinho Agra (30%), Carlos J. Luz (30%), Cristina Requejo (30%), Domingos M. Cardoso (10%) and Tatiana Tchemisova (15%).
- **Expected results** (max. 100 words): Characterization of graphs with convex quadratic stability number. Polynomial time recognition of this type of graphs in particular families. Applications of the results about graphs with convex-QP stability number to the determination of maximum matchings by convex quadratic programming. Improvements on lower and upper bounds on the stability number of graphs determined by convex quadratic programming. Relations between this upper bounds and the Lovasz theta number. Relations between the optimal parameter of a self-concordant barrier over a symmetric cone, the Carathéodory number of the cone and the rank of an underlying Euclidean Jordan algebra and their extensions to combinatorial problems. Optimality conditions for abnormal extrema.
- **Task description** (max. 200 words): Along this task, graphs for which we have not been able to recognize whether they are or not graphs with convex-QP stability number are studied. Particular families, with particular subsets of vertices will be identified. Namely the recognition of these graphs is related to the existence of (k, τ)-regular sets, with $k = 0$ and τ equal to the symmetric value of the minimum adjacency eigenvalue. This research also aims at obtaining effective computable upper and lower bounds for the stability number of graphs leading to the determination of these upper and lower bounds by convex quadratic programming. We study optimal parameters of self-concordant barriers in conic optimization, using Euclidean Jordan algebras, trying to extend the obtained results to combinatorial optimization and graph theory. Abnormality phenomena will be studied from the point of view of geometric control theory where the computation of Morse index and nullity of abnormal extremal plays a crucial role in the verification of the optimality conditions.

1.7.3 Combinatorics and related topics

- **Task duration - months**: 36

- **persons*month**: 27
  - **Research team**: Alexander Plakhov (10%), Domingos M. Cardoso (15%), Eugénio Rocha (10%) e Paula Carvalho (40%).
- **Expected results** (max. 100 words): Construction techniques of strongly regular graphs and results about the existence of the fourth graph of Moore, more specifically: assuming that the fourth graph of Moore exists, to obtain results about the existence of some subsets of vertices with particular regularity properties. Linear and weak-order extensions of quasi-orders defined by weighted sums of binary preference relations in the framework of multiattribute ranking problems and validation techniques for multiattribute ranking solutions. Estimation of the portion of concordance pairs among all possible pairs of linear orders defined over a set X and estimation of the portion of concordance graphs among all graphs with vertex set X. Developments in digital topology with applications of specialization orders on image processing and image recognition.
- **Task description** (max. 200 words): There are several models for the construction of strongly regular graphs, namely by means of integer programming or the definition of a family of partitions of a particular set. The properties of this family will be studied within the framework of poset theory. Special attention will be given to the Moore graphs short family of diameter 2, namely to the possible existence of the fourth graph of Moore. Note that this family has only 3 known graphs (which are strongly regular with valency 2, 3 and 7) and possibly only one more (the fourth graph of Moore, with valency 57). A weak order extension of a quasi-order (that is a reflexive and transitive binary relation) defined by the transitive closure of weighted sum binary preference relations on multiattribute ranking problems can be used for validation of ranking results. Digital topological spaces which are discrete spaces of points, usually defining computer images, combined with specialization orders, become a crucial tool for image processing and image recognition techniques.

1.7.4 Stochastic optimization and stochastic approximation

- **Task duration - months**: 36
- **persons*month**: 61,2
  - **Research team**: Alexander Plakhov (10%), António Batel Anjo (30%), João Pedro Cruz (100%), Paula Oliveira (30%).
- **Expected results** (max. 100 words): Optimization models and algorithms for network problems with random costs on arcs, nodes and both. A stochastic approximation algorithm with geometric step size adaptation is based on the heuristic techniques of step size adaptation known from the literature. It allows one to speed up convergence at the expense of precision of determining the solution.

- **Task description** (max. 200 words): The research on stochastic optimization is based on the study of models for maximizing the expected value of an utility function over a directed random network, where the cost related to the arcs are real random variables following a given distribution. The results on stochastic approximation will be obtained using methods of probability theory, in particular of martingale theory and of stochastic approximation theory.

1.7.5 Nonstandard optimization techniques.

- **Task duration - months**: 36
- **persons*month**: 23,4
  - **Research team**: Vítor Neves (30%), Natália Martins (40%), Ricardo Almeida (50%).
- **Expected results** (max. 100 words): Nonstandard characterizations of Palais-Smale type sequences or nets and their properties. Establishment of properties of internal functions with different kinds of smoothness. Existence and location of critical points of internal functionals. Existence and classification of internal solutions to ODEs with discontinuous linearities. Characterization of submanifolds of finite dimensional Euclidean spaces by means of their approximation by grids of infinitesimal width wherein discretely differentiable functions act. Embeddings of infinite dimensional manifolds in hyperfinite dimensional Euclidean spaces by means of nonstandard hulls of internal finite maps. Peano and Charathéodory type existence theorems for internal ODEs in locally convex spaces.
- **Task description** (max. 200 words): Construction of non-standard analogs to Palais-Smale condition and applications of Mountain Pass like theorems to Differential Equations. Development of a notion of Differentiable Manifold based on the concept of infinitesimal transformation, that encompasses infinite dimensional manifolds modelled on locally convex spaces. Application to Dynamical Systems.