Which graphs are divisor graphs?

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Abstract

For a finite nonempty set S of positive integers, the *divisor graph $G(S)$ of* S has vertex set S and two vertices i and j of $G(S)$ are adjacent if i divides j or j divides i, while the *divisor digraph $D(S)$ of* S has vertex set S and (i, j) is an arc of $D(S)$ if $i|j$. A graph G is a divisor graph if there exists a set S of positive integers such that S is isomorphic to $G(S)$. It is shown that a triangle-free graph is a divisor graph if and only if it is bipartite. Also $GK2$ is a divisor graph if and only if G is bipartite. A vertex v in an oriented graph D is a transmitter if $\operatorname{id} v = 0$, a receiver if $\operatorname{od} v = 0$, and a transitive vertex if $\operatorname{id} v, \operatorname{od} v > 0$ and for every $u \in N^-(v)$ and $m \in N^+(v)$, $(u, w) \in E(D)$. It is shown that a graph G is a divisor graph if and only if there exists an orientation D of G such that every vertex of D is a transmitter, a receiver, or a transitive vertex.