A Spanning Star Forest Model for the Diversity Problem in Automobile Industry

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1 The Spanning Star Forest Model

Consider the *inclusion relation configurations digraph* $\overrightarrow{G}$, where each vertex denotes a cable configuration k_j (cable with a set of active option connections) and each arc $k_i \to k_j$ means that the cable configuration k_i includes the cable configuration k_j (that is, $k_i \supseteq k_j$). As usually, $V(\overrightarrow{G})$ and $A(\overrightarrow{G})$ denote the set of vertices and the set of arcs of $\overrightarrow{G}$, respectively. Assuming that c_i and c_j are the unit production costs of configurations k_i and k_j, respectively, and also that it is expected to sell n_j configurations k_j, each arc $k_i \to k_j$ has the cost $c_{ij} = n_j(c_i - c_j)$. Thus, c_{ij} is the obtained cost if the cable configuration k_j will be supplied by the cable configuration k_i. A star (we deal with) is a digraph with a central vertex and such that every arc of the star (if there is at least one) is directed from the center to a neighbor. Let us denote by

$$st(k_i, \{k_{j_1}, k_{j_2} \dots, k_{j_p}\})$$

the star with central vertex k_i, set of neighbors $N_i = \{k_{j_1}, \dots, k_{j_p}\}$ and arcs $k_1 \to k_{j_1}, k_1 \to k_{j_2}, \dots, k_1 \to k_{j_p}$. Therefore, this star may be denoted just by $st(k_i, N_i)$. A spanning star forest is a set of stars $st(k_{i_1}, N_{i_1}), \dots, st(k_{i_k}, N_{i_k})$ such that $V(\overrightarrow{G}) = \cup_{j=1}^{k} V(st(k_{i_j}, N_{i_j}))$. Such spanning star forest is denoted by $SSF(I)$, where $I = \{i_1, \dots, i_k\}$ (the center index set) and each star $st(k_{i_j}, N_{i_j}))$ is a component of $SSF(I)$. It is assumed that $SSF(I)$ is

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the best spanning star forest (that is, the minimum arc cost sum spanning star forest), among the ones with star centers in I. Therefore, for each $j \in V(\overrightarrow{G}) \setminus I$

$$j \in N_{i^*} \text{ if and only if } c_{i^*j} = \min_{i \in I} c_{ij}.$$

The aim of the optimization project is to determine the minimum arc cost sum spanning star forest, among the spanning star forests with k star centers, with $k_{min} \leq k \leq k_{max}$. Note that the set of star centers of an optimal spanning star forest is the best set of cable configurations to be produced.

2 Example

Let us consider the *inclusion relation configurations digraph* represented in Figure 1, where $k_1, \ldots, k_6$ are the cable configurations to be purchased.

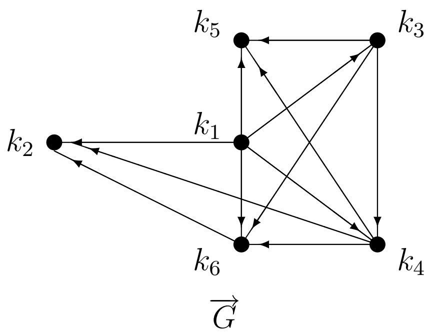

Figure 1: The inclusion relation configurations digraph.

Let us consider the following arc cost matrix C of the digraph $\overrightarrow{G}$, where an entry (i, j) equal to ∞ means that there is no arc between k_i and k_j (that is, the cable configuration k_i does not includes the cable configuration k_j).

$$C = \begin{array}{c} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{array} \begin{array}{c} \begin{array}{cccccc} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 \end{array} \\ \begin{pmatrix} 0 & 4 & 1 & 3 & 1 & 4 \\ \infty & 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & 2 & 3 \\ \infty & 1 & \infty & 0 & 1 & 1 \\ \infty & \infty & \infty & \infty & 0 & \infty \\ \infty & 3 & \infty & \infty & \infty & 0 \end{pmatrix} \end{array},$$

Then, the spanning star forest $SSF(\{k_1, k_3\})$, which is the best spanning star forest among the ones with centers in the set $\{k_1, k_3\}$, has components $st(k_1, \{k_2, k_5\})$ and $st(k_3, \{k_4, k_6\})$. See Figure 2-(b).

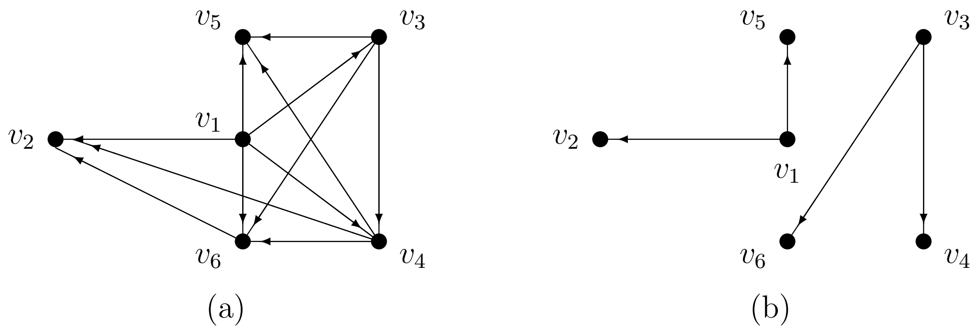

Figure 2: The spanning star forest $SSF(\{k_1, k_3\})$.

The spanning star forest $SSF(\{k_1, k_3\})$ may be represented by the following tableau:

	k_1	k_2	k_3	k_4	k_5	k_6	
	k_1	k_1	k_3	k_3	k_1	k_3	
k_1	$\boxed{0}$	$\boxed{4}$	2	3	$\boxed{1}$	4	5
k_3	∞	∞	$\boxed{0}$	$\boxed{2}$	2	$\boxed{3}$	5
z	0	4	0	2	1	3	10

Table 1: The $SSF(\{k_1, k_3\})$ tableau.

where the most right column contains the costs of the stars $st(k_1, \{k_2, k_5\})$ and $st(k_3, \{k_4, k_6\})$ (which correspondes to the sum of the framed entries), the most left column contains the star centers, each entry of the second upper row contains the star center to which the vertex in the above entry is connected (for instance, if k_i is below k_j, with $i \neq j$, this means that there is an arc (k_i, k_j) in the current spanning star forest). The z-raw entries are the arc costs of the current spanning star forest (that is, the costs of framed boxes). Finally, the lower right corner entry is the arc cost sum of the current spanning star forest. It must be noted that each framed box belongs to the intersection of a table row with a table column, and marks the unique arc from the star center of the row (the cable configuration which appears in the most left entry) to the vertex which appears in the upper column entry.

However, the spanning star forest $SSF(\{k_1, k_3\})$ is not optimal. In fact, the spanning star forest $SSF(\{k_1, k_3\})$ has less arc cost sum. See Figure 3-(b) and Table 2.

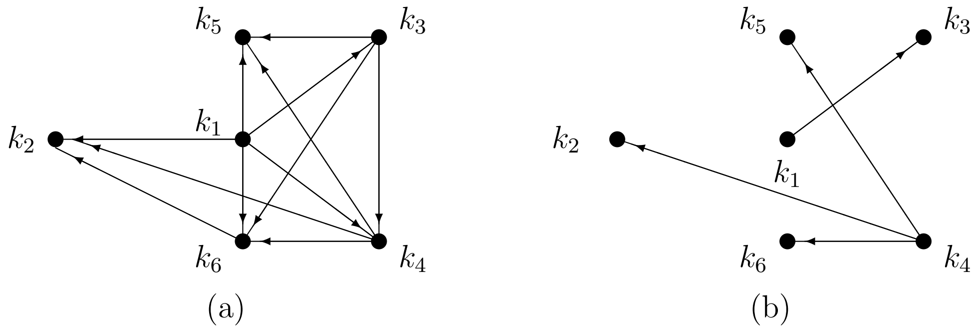

Figure 3: The spanning star forest $SSF(\{k_1, k_4\})$.

	k_1	k_2	k_3	k_4	k_5	k_6	
	k_1	k_4	k_1	k_4	k_4	k_4	
k_1	$\boxed{0}$	4	$\boxed{2}$	3	1	4	2
k_4	∞	$\boxed{1}$	∞	$\boxed{0}$	$\boxed{1}$	$\boxed{1}$	3
z	0	1	2	0	1	1	5

Table 2: The $SSF(\{k_1, k_4\})$ tableau.